A Study of Robustness Between Two Strategic 4D Trajectory Plannings

Daniel Delahaye, Paveen Juntama, Sameer Alam

Proc. of the IWAC2022
pp. XX–XX, Oct. 2022

A Study of Robustness Between Two Strategic 4D Trajectory Plannings

Paveen Juntama[1)†], Sameer Alam[2)], and Daniel Delahaye[1)]

[1)] *Ecole Nationale d'Aviation Civile, Toulouse, France*
[2)] *Nanyang Technological University, Singapore*
[†] *email: paveen.juntama@enac.fr*

Strategic 4D trajectory planning is a promising technology for next-generation air traffic management and systems. Some approaches attempt to satisfy the capacity constraint to reduce traffic congestion, while others aim to reduce potential conflicts between trajectories. This paper investigates two approaches to organizing the real traffic in the French airspace at the strategic level. The first approach minimizes interaction between trajectories, while the second reduces traffic congestion so that the controller maintains the traffic without much effort. The associated optimization problems are formulated and resolved by an approximative approach based on simulated annealing. The departure time perturbation was introduced to study the robustness of the two proposed methods. The evaluation of the robustness is performed by Monte Carlo simulation. According to the results, the strategic deconfliction method completely solved all interactions between trajectories, and the strategic decongestion method reduced traffic congestion by 99.94%. Furthermore, the comparative study shows that the method reducing congestion is more robust against the departure time perturbation than the method minimizing interaction between trajectories. These findings encourage the appropriate use of proposed methods in the strategic 4D trajectory planning framework.

Key Words : traffic deconfliction, traffic decongestion, 4D trajectories, strategic trajectory planning, interaction, air traffic congestion, monte carlo simulation

1. Introduction

The current air traffic management (ATM) system has been dealing with an ever-increasing demand for air travel as a result of economic growth. Although the COVID-19 epidemic has affected current traffic demand since 2020, a recent study[1)] forecasted that European traffic will recover to 2019 levels by the end of 2023 and then climb by 6% in 2027. Moreover, some political events may have tremendous impacts on air traffic. Since February 2022, the airspace closure due to the Russian invasion of Ukraine has caused a wave of cancelled and uneconomic rerouted flights.[2)] Traffic rerouting causes congestion in the adjacent airspace. As capacity limits are reached faster than usual, airspace congestion will expand rapidly, which will result in extra pressure on the network, further delays and cancellations.

This paper proposes two strategic planning approaches to be applied at the strategic level, i.e. several hours or days before real-time operations. The first approach is the strategic deconfliction approach, in which minimizing total interaction between trajectories is the main objective. The second approach is the decongestion approach which satisfies the capacity constraint by mitigating the ATC complexity to which the ATC workload is susceptible. In addition to achieving their primary objectives, these approaches aim to reduce the total departure time shift, route deviation, and flight level shift. A simulated annealing-based resolution algorithm is developed to solve both strategic planning problems.

The main challenge of strategic planning is dealing with a high level of uncertainty, particularly resulting from *uncertain departure times*. The difficulty in synchronizing the activities of different actors at departure airports and the existence of external constraints, e.g., weather conditions, usually produce some uncertainties on the exact time in such a way that the planned strategic 4D trajectories may not be exact. Therefore, the robustness of trajectories becomes important for dealing with disturbances. This paper presents a comparative study concerning the robustness of the two solutions obtained from the proposed approaches. Monte Carlo simulations were performed to evaluate the robustness against the departure time perturbation.

The structure of this paper is as follows: Section 2. provides related work on trajectory-based deconfliction and traffic decongestion methodologies. Section 3. presents the formulation of strategic planning problems. Section 4. describes the proposed resolution algorithm. We discuss the results in section 5., and the paper is finally concluded in section 6..

2. Related work

In this section, the existing strategies to address the strategic trajectory planning problem in the literature are given in the large scale application context. Such strategies include trajectory deconfliction and traffic decongestion methods.

2.1. Traffic Deconfliction

Several studies focus on solving conflicts between aircraft trajectories instead of satisfying the capacity constraint. Numerous studies addressing traffic deconfliction in a TBO environment have been presented in the literature. Durand et al.[3] propose two trajectory maneuvers: modifying the heading and the flight level. En-route conflicts between trajectories are solved by the genetic algorithm (GA). Combining ground holding and flight level allocation is given in Ref. 4). Dougui et al.[5] suggested a Light Propagation Algorithm (LPA) based on various light refractions. Certain potential conflicts are solved using a Branch-and-Bound (B&B) algorithm. However, the results in these studies present unsolved conflicts in the large scale context.

Finally, conflict-free trajectories can be achieved using a combination of hill climbing and the simulated annealing algorithm to solve the strategic planning problems at the continental scale.[6,7] Trajectory actions consist of changing the departure time, the horizontal route, and the flight level to resolve the total interaction between trajectories. This paper proposes a strategic deconfliction approach based on such actions. In addition to minimizing interaction between trajectories, the approach aims to minimize total departure time shift, route deviation, and flight level shift.

2.2. Traffic Decongestion

Several research studies have been done to minimize air traffic congestion. In order to adapt the demand to the available capacity, similar to the traffic deconfliction, the congestion is expected to be reduced by moving departure times of flights, changing the current flight routes, and selecting new flight levels.

Historically, traffic assignment methods have been developed to reduce congestion in air traffic networks by spreading the traffic demand in time and space. The ground holding approach in Ref. 8) has been first investigated to regulate traffic demand as a function of the airspace capacity. Later, the rerouting option has been investigated, at a macroscopic level, in Refs. 9,10). However, all the previous studies aim to reduce the number of aircraft within a period of time.

Several works have paid more attention to consideration of ATC workload to reduce airspace congestion. Previous studies in Refs. 11, 12) attempt to reduce the airspace congestion in terms of workload induced in a control sector. These studies present a flow modelling of the traffic network and solve the route-time allocation problem using the genetic algorithm.

As the strong relationships between ATC complexity and ATC workload were examined in Ref. 13), much of the previous research on traffic planning to reduce congestion with respect to traffic complexity has been explored. In Ref. 14), the convergence metric is used to evaluate the ATC complexity between aircraft trajectories. Congestion reduction is made by temporarily creating local route networks in a specific area. Juntama et al.[15] attempt to minimize air traffic complexity using a distributed metaheuristic approach at the strategic level. The traffic complexity between trajectories is measured by the König metric. However, these indicators are primarily based on geometrical properties of traffic. Such indicators would fail when a traffic situation has a more complex organization.

To overcome this issue, this paper proposes strategic decongestion planning in which the evaluation of airspace congestion is developed from the metric based on linear dynamical systems. As introduced in Ref. 16), the metric can quantify the disorder in various kinds of traffic situations at a given time. Like the strategic deconfliction planning, this planning applies the same traffic structuring method: departure time adjustment, route assignment, and flight level allocation. The approximation approach based on simulated annealing is developed from the previous work in Ref. 15) to solve both problems. This approach is able to improve the computation performance without sacrificing the quality of the final solution.

One of the main challenges is considerable uncertainty regarding traffic demand. Perturbation of aircraft trajectories in the time dimension has been presented to evaluate the robustness in Refs. 17–19). In this paper, we investigate the robustness against perturbed flight departure times between the strategic deconfliction planning and the strategic decongestion planning. The Monte Carlo method is used to simulate the different sets of traffic from the solutions of both plannings. As such traffic is randomly perturbed, we measure its additional interactions to evaluate the robustness.

3. Problem formulation

This section establishes the mathematical framework of the proposed strategic planning methodology. First, given data regarding this problem is present. Then, the set of decision variables and their constraints are given. Finally, two objective functions are described at the end of the section.

3.1. Input data

Assume there is a set of flights, $\mathcal{F}$ on a given day. For each flight $i \in \mathcal{F}$, the following data is given as follows:

- A set of initial 4D aircraft trajectories;
- γ_i: the trajectory of flight i;
- τ_i^+: the maximum allowed delay departure time shift for flight i;
- τ_i^-: the maximum allowed advance departure time shift for flight i;
- r_i^+: the maximum allowed alternative routes for flight i;
- l_i^+: the maximum allowed positive flight level shift for flight i;
- l_i^-: the maximum allowed negative flight level shift for flight i;

- N_h: the required horizontal seperation;
- N_v: the required vertical seperation.

3.2. Decision variables

The three following decision variables are used to structure the aircraft trajectories: departure time adjustment, route assignment, and flight level allocation.

Departure time adjustment The departure time of each flight can be rescheduled with a positive (delay) or a negative (advance) time shift. Let τ_i be the time shift assigned to flight i. Therefore, the new departure time of flight i is therefore $t'_i = t_i + \tau_i$ where t_i is the original departure time of flight i indicated in the initial flight plan.

Alternative en-route trajectory The horizontal route of each flight can be changed with one of the alternative routes predefined for each flight. These alternative routes are all generated by a BADA-based fast time simulator. The construction of these routes relies on the deviation from their original routes. Let $r_i \in \mathbb{N}_0$ be the current route index of flight i. Figure 1 illustrates possible alternative horizontal profiles of flight i.

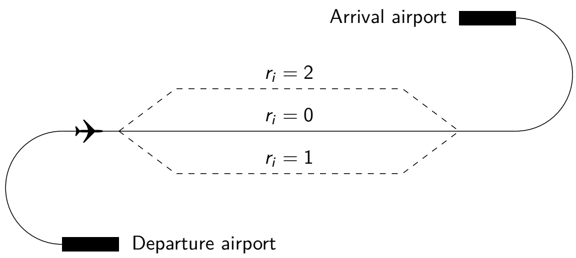

Fig. 1: Alternative horizontal profiles (the dashed line) based on an original route (the solid line) of flight i.

Flight level allocation The final choice to structure the traffic is to assign the new flight level with a flight level shift $l_i \in \mathbb{Z}$. An example of the vertical profile for each flight is presented in Fig. 2. Hence, the new flight level h'_i is $h'_i = h_i + l_i$.

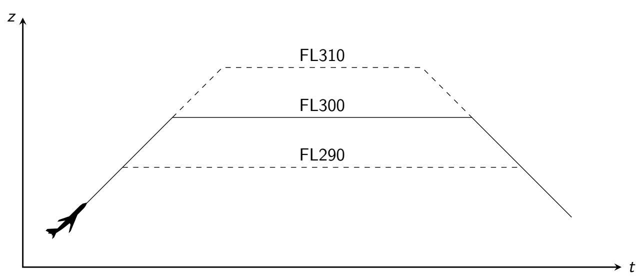

Fig. 2: Alternative vertical profiles (the dashed line) based on an original flight level (the solid line) of flight i.

To simplify the proposed model, the decision vectors of departure time shifts, route options, and flight level shifts are respectively defined as follows:

$$\boldsymbol{\tau} = \{\tau_i : \tau_i \in \mathbb{Z}, \forall i \in \mathcal{F}\}$$
$$\mathbf{r} = \{r_i : r_i \in \mathbb{N}_0, \forall i \in \mathcal{F}\}$$
$$\mathbf{l} = \{l_i : l_i \in \mathbb{Z}, \forall i \in \mathcal{F}\}$$

Therefore, the decision variables of our problem are represented with the following single vector:

$$\mathbf{u} := (\boldsymbol{\tau}, \mathbf{r}, \mathbf{l})$$

3.3. Constraints

The preceding decision variables of flight i shall be compliant with the following constraints: the maximum departure time shifts, the maximum index of route options, and the maximum flight level shifts. Such constraints are defined as follows:

$$\tau_i^- \leq \tau_i \leq \tau_i^+, \quad \forall i \in \mathcal{F} \tag{1}$$
$$0 \leq r_i \leq r_i^+, \quad \forall i \in \mathcal{F} \tag{2}$$
$$l_i^- \leq l_i \leq l_i^+, \quad \forall i \in \mathcal{F} \tag{3}$$

3.4. Objective functions

This section presents the following objective functions: the interaction between trajectories for the strategic deconfliction method and air traffic complexity based on linear dynamical systems for quantifying levels of traffic congestion in the strategic decongestion method.

Interaction between trajectories Regarding strategic planning in a TBO environment, the interaction between trajectories indicates when two or more trajectories occupy the same space at the same period of time. A more clarified definition of this concept is given in Refs. 6, 7).

Considering a given set of discretized 4D trajectories, where each trajectory γ_i represents a time sequence of 4D points, each 4D point, $P_{i,k} = (x_{i,k}, y_{i,k}, z_{i,k}, t_{i,k})$ specifies that the aircraft must arrive at a given position $(x_{i,k}, y_{i,k}, z_{i,k})$ at time $t_{i,k}$ for $k \in \mathcal{K}_i$, and $\mathcal{K}_i$ is the set of sequence numbers obtained from the discretization of the trajectory γ_i.

For any pair of points $P_{i,k}$ and $P_{j,l}$ at time $t_{i,k}$ on the trajectories γ_i and γ_j, respectively, a potential conflict of such trajectories can occur when the required separation is violated as follows:

$$d_h(P_{i,k}, P_{j,l}) < N_h \tag{4}$$
$$d_v(P_{i,k}, P_{j,l}) < N_v \tag{5}$$

When preceding conditions are satisfied, the definition that the point $P_{i,k}$ is in conflict with the point $P_{j,l}$ at the same time is given by:

$$\mathcal{C}(P_{i,k}, P_{j,l}) := \begin{cases} 1, & \text{if Point } P_{i,k} \text{ is in conflict with Point } P_{j,l} \\ & \text{under conditions (4) and (5),} \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Considering at time $t_{i,k}$ of trajectory γ_i, let $\Phi_{i,k}$ be the number of interactions at point $P_{i,k}$. It is defined as the number of times that a new potential conflict (as defined in Eq. (6)) could be detected involving $P_{i,k}$. Hence, $\Phi_{i,k}$ is given by:

$$\Phi_{i,k}(\mathbf{u}_i) = \sum_{\substack{j \in \mathcal{J} \\ j \neq i}} \sum_{l \in \mathcal{K}_j} \mathcal{C}(P_{i,k}, P_{j,l}) \tag{7}$$

where $\mathcal{K}_j$ is a set of points along with each trajectory j at time $t_{i,k}$, and $\mathcal{J}$ denotes a set of neighboring trajectories in the search space.

The number of interactions associated with the trajectory γ_i, Φ_i is therefore defined as follows:

$$\Phi_i(\mathbf{u}_i) = \sum_{k \in \mathcal{K}_i} \Phi_{i,k}(\mathbf{u}_i) \tag{8}$$

Finally, the number of interactions between all trajectories over a full-time horizon is defined as

$$\Phi(\mathbf{u}) = \sum_{i \in \mathcal{F}} \Phi_i(\mathbf{u}_i) = \sum_{i \in \mathcal{F}} \sum_{k \in \mathcal{K}_i} \Phi_{i,k}(\mathbf{u}_i) \tag{9}$$

In addition, a practical methodology to evaluate interaction between trajectories in a large-scale context is presented in Section 3.5..

Air Traffic Complexity based on Linear Dynamical System The metric is adapted to evaluate traffic congestion over a full-time horizon in this paper. Unlike straightforwardly counting the number of aircraft, aircraft positions and speed vectors are used to measure the ATC complexity associated with a given traffic situation. As a result, the metric is able to quantify congestion between trajectories in the airspace. Some identical traffic situations were given and tested with this metric in Ref. 16).

Given that a set of trajectories is discretized in time into a 4D airspace, each trajectory γ_i represents a set of observations in time series. Each observation at time $t_{i,k}$ can be represented by the position and speed vectors, $\mathbf{x}_{i,k}$ and $\mathbf{v}_{i,k}$, respectively, in the following matrix form:

$$\mathbf{Z}_{i,k} = \begin{bmatrix} \mathbf{x}_{i,k} & \mathbf{v}_{i,k} \end{bmatrix}^\top \tag{10}$$

where $\mathbf{x}_{i,k} = \begin{bmatrix} x_{i,k} & y_{i,k} & z_{i,k} \end{bmatrix}^\top$ and $\mathbf{v}_{i,k} = \begin{bmatrix} vx_{i,k} & vy_{i,k} & vz_{i,k} \end{bmatrix}^\top$. To compute the local congestion of a given traffic situation, it is necessary to formulate such a situation into a linear dynamical system. The traffic situation involving the trajectory i at time $t_{i,k}$ represents the current observation $\mathbf{Z}_{i,k}$ on the trajectory i, plus a set of observations lying on trajectories of neighboring aircraft that is present in the vicinity of the reference aircraft i at position $\mathbf{x}_{i,k}$. Accordingly, the set of observations presented in the neighborhood search space is defined by:

$$\mathcal{N}_{i,k} = \{\mathbf{Z}_{j,k'} : \mathcal{V}(i,k,j,k') = 1, t_{j,k'} = t_{i,k}, \\ k' \in \mathcal{K}_j, j \in \mathcal{F} \setminus \{i\}\} \tag{11}$$

where $\mathcal{V}(i,k,j,k')$ indicates if aircraft j is in the neighborhood search space at time $t_{i,k}$. When such observations are obtained, a linear dynamical system can be formulated.

For the sake of simplicity, given that an observation is represented by a position measurement, $\mathbf{x}$ and a speed measurement, $\mathbf{v}$. A linear dynamical system is controlled by the following equation:

$$\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{b} \tag{12}$$

where $\dot{\mathbf{x}}$ is the speed vector associated with each point in the state space, $\mathbf{x}$ is the position vector, $\mathbf{A}$ and $\mathbf{b}$ represents the static behavior of the system.

To determine an accurate dynamical system model best fitted to observations in the state space, it is necessary to find the matrix $\mathbf{A}$ and vector $\mathbf{b}$, which minimizes the error between speed observations and estimated speed vectors. Such a minimization problem can be formulated as follows:

$$\mathbf{A}^*, \mathbf{b}^* = \underset{\mathbf{A},\mathbf{b}}{\operatorname{argmin}} \sum_{n \in \mathcal{N}} \|\mathbf{v}_n - (\mathbf{A}\mathbf{x}_n + \mathbf{b})\|^2 \tag{13}$$

where $\mathcal{N}$ is the set of observations in the state space (e.g., a given traffic situation). The calculation of the matrix $\mathbf{A}^*$ and vector $\mathbf{b}^*$ is detailed in Ref. 16).

When the matrix $\mathbf{A}_{i,k}$ is derived for the traffic situation of the aircraft i at time $t_{i,k}$, extraction of the eigenvalues from $\mathbf{A}_{i,k}$ is required for determining the local congestion $\Psi_{i,k}$. Let $\lambda_{i,k}^{(1)}$, $\lambda_{i,k}^{(2)}$, and $\lambda_{i,k}^{(3)}$ be the three complex eigenvalues of $\mathbf{A}_{i,k}$, the evolution of the traffic situation is determined by such eigenvalues. The eigenvalues with real positive values correspond to a divergent situation where aircraft fly in expansion mode. On the other hand, when the real parts are negative, aircraft fly in a convergent situation. Furthermore, when the real parts of such eigenvalues are null, the associated traffic situation is fully organized in parallel or rotation.

According to these properties, the local congestion metric is developed based on the intensity of the convergence tendency in the traffic situation. When the traffic is well organized, the congestion becomes null. It must be noticed that if the resulting behaviour represents a divergent motion, such a motion would be considered an organized pattern because it will not affect the controller's workload since the divergent aircraft will not cause any potential conflict. Therefore, the local congestion $\Psi_{i,k}$ can be calculated by using the following equation:

$$\Psi_{i,k} = \sum_{\mathrm{Re}(\lambda_{i,k}^{(n)})<0} \left|\mathrm{Re}(\lambda_{i,k}^{(n)})\right|, \quad n \in \{1,2,3\} \qquad (14)$$

To compute the summation of the local congestion along the trajectory γ_i, we can determine Ψ_i as follows:

$$\Psi_i = \sum_{k\in\mathcal{K}_i} \Psi_{i,k} \qquad (15)$$

Finally, the traffic congestion Ψ can be obtained by accumulating congestion values for all trajectories in the airspace, as expressed in the following equation:

$$\Psi = \sum_{i\in\mathcal{F}} \Psi_i = \sum_{i\in\mathcal{F}} \sum_{k\in\mathcal{K}_i} \Psi_{i,k} \qquad (16)$$

Therefore, we proposes two optimization problems to perform the comparative analysis of different strategic 4D trajectory plannings. First, in this paper, the objective of the strategic traffic deconfliction is to minimize interaction between trajectories while minimizing the total departure time shift, deviation from the nominal routes and flight level shift. The optimization problem can be formulated as follows:

$$\begin{aligned} \min_{\mathbf{u}} \quad & J_1(\mathbf{u}) = \Phi(\mathbf{u}) + \alpha_1 \sum_{i\in\mathcal{F}} \tau_i + \eta_1 \sum_{i\in\mathcal{F}} l_i \\ & \qquad + \beta_1 \sum_{i\in\mathcal{F}} d_A\left(\gamma_i(r_i), \gamma_i(0)\right)^2 \\ \text{s.t.} \quad & \tau_i^- \le \tau_i \le \tau_i^+,\ \forall i \in \mathcal{F} \\ & 0 \le r_i \le r_i^+,\ \forall i \in \mathcal{F} \\ & l_i^- \le l_i \le l_i^+,\ \forall i \in \mathcal{F} \end{aligned} \qquad (17)$$

Second, the strategic traffic decongestion problem in this paper aims to mitigate the congestion in air traffic while minimizing the total departure time shift, deviation from the nominal routes and flight level shift. This problem can be expressed in the following mathematical form:

$$\begin{aligned} \min_{\mathbf{u}} \quad & J_2(\mathbf{u}) = \Psi(\mathbf{u}) + \alpha_2 \sum_{i\in\mathcal{F}} \tau_i + \eta_2 \sum_{i\in\mathcal{F}} l_i \\ & \qquad + \beta_2 \sum_{i\in\mathcal{F}} d_A\left(\gamma_i(r_i), \gamma_i(0)\right)^2 \\ \text{s.t.} \quad & \tau_i^- \le \tau_i \le \tau_i^+,\ \forall i \in \mathcal{F} \\ & 0 \le r_i \le r_i^+,\ \forall i \in \mathcal{F} \\ & l_i^- \le l_i \le l_i^+,\ \forall i \in \mathcal{F} \end{aligned} \qquad (18)$$

where $d_A(\gamma_i(r_i), \gamma_i(0))^2$ denotes the area-based distance between the nominal and alternative trajectoriesof flight i that results from the distance integration over time and the evaluation of a mean error instead of the raw sum of squares.[20)]

3.5. Interaction Detection

To calculate the number of interactions between all trajectories in the airspace, a grid-based computation approach is implemented by using a hash table as presented in Refs. 6, 7, 15).

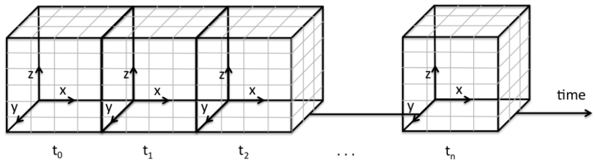

Fig. 3: 4D (space-time) grid.

The airspace is discretized into a 4D grid (3D space + time), as shown in Fig. 3. The size of each cell in the spatial dimension is defined by horizontal and vertical separation requirements, N_h, N_v, and the time axis is scaled based on a sampling time present in a set of trajectories. All trajectories are stored in such a 4D grid, whereby each aircraft plot is inserted into each cell represented by an array of the hash table. This table allows the algorithm to retrieve or manipulate the associated plot from the 4D grid with the average time complexity of $\mathcal{O}(n) = 1$.

To calculate the number of interactions around a reference plot $P_{i,k}$ at time $t_{i,k}$, we search for candidate plots that belong to other aircraft in the same cell and adjacent cells corresponding to the time $t_{i,k}$. Then, we calculate the horizontal and vertical distances between the reference and each candidate plot. Finally, when the candidate plot violates either one or both minimum separations, the interaction is computed using Eq. (6). Specifically, the interaction will be computed based on the reduced lateral minimum separation in the TMA zone. However, an interaction may not be able to be detected during a given sampling time. To avoid this issue, an interpolation technique can be used to construct temporary plots with a sufficiently small step. More details of this technique are presented in Ref. 6).

3.6. Congestion Computation

Like the interaction detection scheme, we use a 4D grid to store, retrieve and manipulate 4D trajectories in which aircraft position and speed vectors are represented in 4D cells.

Considering a traffic situation around the reference aircraft i at position $\mathbf{x}_{i,k}$, the following observations corresponding to the time $t_{i,k}$ are declared candidates for neighborhood filtering: 1) the observation of the reference aircraft; and 2) the observations situated in the cells adjacent to the cell in which the observation of the reference aircraft is located. The operation of neighborhood filtering in x, y, and z dimensions is detailed in Ref. 16). Accordingly, the candidates validated by the neighbor filtering process shall be taken into account in computing the local congestion based on Eqs. (13) and (14).

4. Resolution Algorithm

Given that the state space of the optimization problem can be represented as a set of individual decisions $\mathcal{D}$ in which each decision associated with each flight can be changed in order to generate a neighborhood solution. Figure 4 displays a vector of decisions and their associated cost values obtained by evaluation-based simulation. When a neighborhood solution was generated, all decisions are required for a simulation to determine a new objective value since changing a decision may impact to other cost values. Such a simulation may require excessive memory, which leads to an exhaustive computation time.

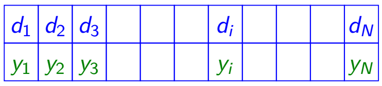

Fig. 4: Vector of decisions with their performance values

This paper proposes a selective simulated annealing (SSA) algorithm to evaluate a current decision in a given iteration instead of evaluating all decisions. However, the SSA evaluates all decisions in a certain iteration to ensure the coherence of the overall objective value. Like the conventional SA, the algorithm takes a new decision to generate a neighborhood solution randomly, measures its quality, and moves to it with respect to the temperature-dependent probabilities of selecting better or worse solutions. The SSA algorithm is adapted to our problem with the following configurations.

Neighborhood function The neighborhood function generates a candidate decision by the following two steps:

1. A decision with a higher cost value is more likely to be chosen for generating a candidate decision. Therefore, the algorithm chooses a decision with the following selective probability function:

$$P_s(y_i) = a + (1-a) \cdot \left(\frac{y_i}{y_{\max} - y_i} \right)^b \tag{19}$$

   where $y_{\max}$ is *the highest cost value* in a set of all decisions at a given transition. The constants a and b are the user-defined parameters of the selective probability function where $0 \le a \le 1$ and $b > 0$.

2. As illustrated in Fig. 5, the current decision d_i represents the decision variables associated with aircraft i. One or more decision variables are randomly modified inside boundaries defined by Eqs. (1), (2), and (3).

The probability of choosing how trajectory γ_i is modified depends upon the ratio of the initial temperature T_0 and the current temperature T in a given transition. The neighborhood function in this paper is summarized in Algorithm 1.

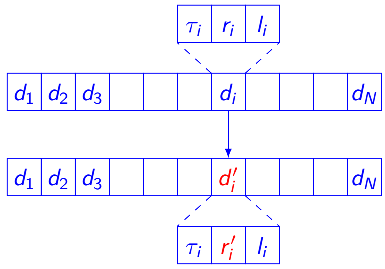

Fig. 5: Neighborhood generation

Algorithm 1 Neighborhood function

Require: trajectory i, initial temperature T_0, current temperature T

procedure CHANGE_DECISION(T_0, T)
  Generate a random number, $p := random(0, 1)$
  if $p < \frac{T}{T_0}$ **then**
    Choose randomly new r_i or l_i or both.
  else
    Choose randomly new τ_i.

Initial temperature We perform the heating process to determine the initial temperature T_0 in this work. The process involves applying a decision change and its cost evaluation for each iteration. The difference cost from changing decision is used to accept or reject a new decision with the following Metropolis-based criterion:

$$Pr\{\text{accept } d_i'\} = \begin{cases} 1, & \text{if } y_i' \ne y_i, \\ \exp\left(\dfrac{y_i' - y_i}{T}\right), & \text{otherwise.} \end{cases} \tag{20}$$

where T is the overall temperature. This temperature increases until the initial acceptance rate goes beyond a configurable threshold χ_0.

Furthermore, we rely on a geometrical cooling schedule for which the evolution of the temperature T_k is given by the following function: $T_{k+1} := \alpha T_k$. We use the fixed number of iterations N_I at each temperature in order to ensure that an equilibrium state is reached at each temperature step. Finally, the SSA algorithm stops and returns the final solution when the final temperature T_f reaches the value $\epsilon_s \cdot T_0$ so that the probability of acceptance is sufficiently small where $0 < \epsilon_s \le 1$.

5. Results

The proposed resolution algorithm and Monte Carlo method are implemented in Java, and all experiments are run on an Ubuntu system with Intel Xeon at 2.4 GHz with 32 GB of memory. It is tested with two different strategic planning models. The experiment's air traffic data represents the en-route traffic in the French airspace. It consists of 8,476 trajectories represented by 21,371,854 sample 4D points, including alternative trajectories. Table 1 provides the parameter values that define the problem. The parameters used for the resolution algorithm have been conducted by several empirical experiments and are separately defined in Table 2.

Table 1: User-defined parameters corresponding to the problem formulation.

Parameters	Value
Sampling time step, t_s	15 s
Maximum negative departure time shift, $\tau^-_{\max}$	15 min
Maximum positive departure time shift, $\tau^+_{\max}$	45 min
Maximum number of route options, $r^+_{\max}$	2
Maximum negative flight level shift, $l^-_{\max}$	2
Maximum positive flight level shift, $l^+_{\max}$	2
Objective function coefficients for the strategic deconfliction problem, $(\alpha_1, \beta_1, \eta_1)$	(0.005, 1, 0.25)
Objective function coefficients for the strategic decongestion problem, $(\alpha_2, \beta_2, \eta_2)$	$0.01 \cdot (\alpha_1, \beta_1, \eta_1)$

Table 2: User-defined parameters corresponding to the resolution algorithm.

Parameters	Value
Number of iterations at each temperature step, N_I	1000
Constants for the selective probability function, (a, b)	$(0.05, 3)$
Initial acceptance rate, χ_0	0.8
Geometric cooling rate, β	0.999
Final temperature, T_f	$10^{-4} \cdot T_0$

The strategic deconfliction problem is solved by the SSA algorithm. The initial and final interaction between trajectories, total delay time, route deviation factor, and flight level shifts are reported in Table 3. The resolution algorithm reaches an interaction-free trajectory plan within 30 min from 89.88 min. The SSA algorithm with the same configuration is used to solve the strategic decongestion problem. The initial and final congestion between trajectories, total delay time, route deviation factor, and flight level shifts are reported in Table 4. The proposed method for which the total computation time is 85.8 min can reduce the congestion between trajectories by 99.94%. The strategic decongestion planning method generates more of the average departure time shift and flight level shift than the traffic deconfliction method. However, there is not much difference in the average route deviation.

Table 3: Numerical results obtained by the strategic deconfliction method.

Initial Φ	Final Φ	Solved interactions	Avg. time shift (min)	Avg. route deviation	Avg. flight level shift
119354	0.0	100.00%	2.56	0.2867	1.24

Table 4: Numerical results obtained by the strategic decongestion method.

Initial Ψ	Final Ψ	Solved congestion	Avg. time shift (min)	Avg. route deviation	Avg. flight level shift
1918.2	1.18	99.94%	4.84	0.2871	1.46

Finally, we employ the Monte Carlo simulations to evaluate the robustness of two strategic planning methods with respect to the departure time perturbation. This study uses optimal trajectory plans from these planning methods. To produce the departure time perturbation, the scheduled departure times of flights are randomly adjusted by a normal distribution so that the number of interactions between trajectories potentially increases. In our assumption, the departure time of each flight is varied by a normal distribution with a mean departure delay of 5 min over the time interval $[-50, 60]$ min. The experiment consists of ten Monte Carlo simulations with different numbers of perturbed flights. Each simulation runs 10,000 replications to achieve the statistical analysis.

After simulation, we measure the additional numbers of interactions from the two different sets of perturbed traffic. Let Δy_n be the additional number of interaction between trajectories for each plan n. Figure 6 shows how many additional interactions increased over the number of perturbed flights resulting from 4D trajectory plans. Both plans show similar profiles, with additional interactions increased over the number of perturbed flights. The number of additional interactions for the traffic based on the plan to reduce the congestion is less than that for the traffic based on the plan to minimize interaction between trajectories.

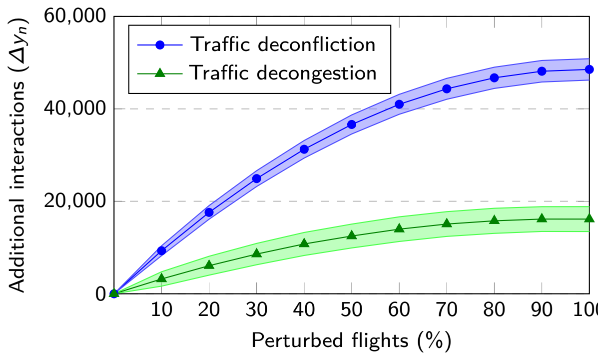

Fig. 6: Number of additional interactions after the departure time perturbation over the number of perturbed flights.

Figure 7 compares the additional number of interactions for both plannings when all flights are perturbed. Although there are no statistically significant variations in the standard deviation, it is evident that the decongestion-based solution outperforms the deconfliction-based solution, which has fewer additional interactions.

6. Conclusions

In this paper, we present two approaches to address a strategic 4D trajectory planning problem. The first approach minimizes interaction between trajectories, while the second approach minimizes congestion between trajectories. Behind proposed optimization models, both methods also aim at reducing total

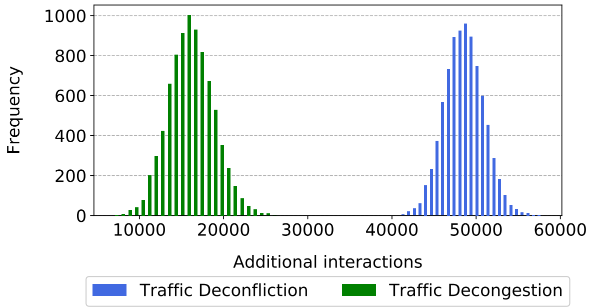

Fig. 7: Frequency distributions in terms of additional number of interactions when all flights are perturbed. The simulation results are obtained from the Monte Carlo simulations based on the solutions of the deconfliction planning and the decongestion planning.

departure time delay, route deviation, and flight level shift. The selective simulated annealing algorithm is proposed to solve the strategic planning problem for the en-route traffic in the French airspace. The results show that the resolution algorithm can solve all interactions between trajectories for the strategic deconfliction model. The same algorithm gives the optimal trajectory plan with a 99.94% reduction of airspace congestion for the strategic decongestion model.

This paper also presents a comparative study regarding the robustness of the proposed methods against the departure time perturbation, whereby their solutions are evaluated through Monte-Carlo simulations. However, the perturbation of flight time between two 4D points has not been taken into account in this work. According to the results, the method to minimize traffic congestion is more robust against the disturbances than the method to minimize interaction between trajectories. This comparative study can support ATFM planners in using appropriate methods in the strategic 4D trajectory planning framework to deal with a high level of uncertainty.

Acknowledgments

This work has been supported by French Government Scholarship (BGF). The author gratefully acknowledges Julien Lavandier for his assistance in data acquisition and cleaning and the late Serge Roux for his kind support with system administration.

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