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Abstract

This paper proposes a study motivated by the problem of minimizing the environmental impact of air transport at the level of a complete air network, considering thereby several aircraft. Both CO_2 and non-CO_2 effects are taken into account to calculate this impact. The proposed methodology takes into account a network point of view where airspace capacities evolve as well as the traffic itself over time. Finding a shortest path with various constraints and cost functions is a common problem in operations research. This particular study deals with the special case of computing multiple shortest paths with capacity constraints on a time-dependent subgraph. Multiple shortest paths is understood as one shortest path for each vehicle considered. The static special case is modeled as a Mixed Integer Linear Program (MILP), so that it can be solved directly by a standard commercial solver. The time-dependent nature of the problem is then modeled thanks to a sliding-window approach. Encouraging numerical results on the CO_2/contrail-safe aircraft trajectories application are obtained and show that the environmental impact can be significantly reduced while maintaining safety by satisfying the airspace capacity constraints.

Keywords: Subgraph constrained time-dependent shortest path problem, Contrails, Air transportation, Mixed Integer Linear Programming.

1 Introduction

Transportation, and in particular air transportation, offers an important source of operations research problems, such as shortest path under constraints. Motivated by the question of more eco-responsible transport and in a sustainable development approach, these problems generally focus on the minimization of environmental costs. In particular, for air transport, driven by a global will of the sector, the issues of green operations are gaining importance.

The air transportation environmental impact is not solely due to CO_2 effects. The non-CO_2 effects are other greenhouse effect gases such as nitrogen oxide (NO_x) or other more complex phenomena like condensation trails (named contrails hereafter). Contrails form white trails at the back of the aircraft engines under certain conditions of humidity and temperature. They can disappear very quickly or on the contrary persist and turn into clouds, called cirrus [1]. These can cause a parasol effect, usually during the day, preventing the solar rays from reaching the earth. On the contrary, they can also create a greenhouse effect, by reflecting the rays emitted by the earth. In total, despite the important uncertainties that remain, contrails have a negative effect on the climate [2].

Avoiding contrails therefore requires aircraft to avoid certain areas that are too humid and/or too cold. However, this should not be done at the expense of flight safety. This study proposes to plan flights several hours in advance, on a strategic scale. At this scale, ensuring safety means satisfying airspace capacity constraints. Indeed, the airspace is divided into sectors under the responsibility of one or several air traffic controllers. Thus, by limiting the number of aircraft per sector per time period, it is understood that the air traffic controllers workload will be acceptable to ensure safety. On a shorter time scale, a few minutes before arriving in an airspace sector for example, the air traffic controllers ensure safety by enforcing vertical and horizontal separations between aircraft.

To satisfy airspace capacity constraints, two approaches are possible: an airline-centered or a network-centered approach. On the one hand, flight operators propose several flight plans and the network manager[1] looks for a valid and best potential combination. On the other hand, the computations are done directly by the network manager, assimilated in the sequel to the air traffic control. This network-centered approach allows one to reach an optimum, to avoid more cases without a solution satisfying the capacity constraints and to evaluate the consequences of politics to avoid the negative impact of contrails. This approach is the one chosen in this paper.

The considered problem is close to the classical operations research issue of finding shortest paths, which includes several types of problems. A first example is the well-known standard shortest path problems which can be solved by efficient polynomial-time algorithms. There are several extensions of this classical problem, for instance, the time-dependent shortest-path problem with dynamic costs and constraints. Variants involving constraints are NP-hard problems even when dealing with a single vehicle. Other problems deal with several vehicles simultaneously. The problem under study is

[1] National or international entity that aims to ensure the best possible use of the airspace according to its capacities by adapting the traffic. In Europe, the network manager is Eurocontrol. In the United States, the equivalent is the Air Traffic Control System Command Center (ATCSCC).

at the intersection of these mentioned problems: one seeks paths for several vehicles, under constraints, and with dynamic costs and constraints.

The main contribution of this paper is the proposition of a network approach for contrail mitigation. This low environmental-impact aircraft trajectory application comes with a realistic illustrative instance, that is made publically available, and preliminary numerical experiments that show that contrails can be mitigated at the network scale. First, an optimization model first is proposed for the static case. It computes *simultaneously* (not sequentially) a path for each vehicle considered that satisfies capacity constraints on subgraphs. A sliding-window methodology is then proposed to address time-dependent costs and constraints.

This paper proposes first in Section 2 a literature review of individual route optimization for contrails mitigation and route optimization for several vehicles. Then, it presents in Section 3 an optimization model for the static case and its extension to the time-dependent problem. The application to the contrail-avoidance aircraft trajectory problem is addressed in Section 4. Promising numerical experiments are shown and discussed in Section 5 through a sensitivity analysis of the different parameters involved. Section 6 presents general conclusions and perspectives. Appendix A gives the time-discretized optimization model, and Appendix B details how the application input data are computed.

2 Previous related works

This section presents previous related works by focusing first, in Subsection 2.1, on individual route optimization for contrails mitigation. Then, in Subsection 2.2, different works done on assigning routes for several vehicles are presented.

2.1 Individual route optimization for contrail mitigation

In recent years, driven by various initiatives, the issue of green aviation has become more prominent in the literature. For instance, finding trajectories with the least possible CO_2 emissions is a topic well represented in the literature in particular by proposing studies calculating the optimal wind trajectories (see for instance [3–5]). In addition, non-CO_2 effects are a particularly important topic. Several methods taking different points of view and with different resolution strategies have been developed.

To solve the problem in the most general case, optimal control methods have been implemented. Since the constraints related to the mechanics of flight are enforced, the computed trajectory is flyable in the free space. The methods chosen differ according to the dimension of the instance addressed, the objective function considered, and the number of aircraft involved. In [6], the problem is solved in 2D, thanks to optimal control by minimizing an objective function that takes into account contrail avoidance, fuel and flight time. It has been used on one-aircraft instances but also for trajectories between 12 city pairs. Hartjes *et al.* [7] solve the problem in 3D for a single aircraft, also by minimizing fuel, time and time in contrail areas. Some other papers present methods taking into account time, such as [8].

Other studies rely on metaheuristics to solve the problem. For instance, Yin *et al.* [9] use genetic algorithms to compute transatlantic flight trajectory to mitigate the impact of contrails. Methods based on graphs are also used, like A* in [10] or Dijkstra's algorithm [11].

Finally, other methodologies rely on MILP [12] or on Mixed Integer Quadratic Programming (MIQP) [13] formulations.

Simorgh *et al.* remark in [14] that Air Traffic Management (ATM) considerations are less often taken into account when multiple aircraft are considered, although the impact on airspace capacity and controller workload is certain. Indeed, a risk is to empty the spaces favorable to contrails by strongly congesting adjacent airspaces. Addressing problems that aim at avoiding such situations is one of the main contributions of this paper.

2.2 Assigning routes for several vehicles

Finding a shortest path is a common problem in operations research. The literature presents various way to solve such problems. The graph version of the problem can be addressed by integer linear programming [15], or other efficient algorithms such as Dijkstra's algorithm [16], A* [17] or Bellman's algorithm [18] (dynamic programming). In some cases, optimal control techniques [19] can be used, especially if the path is to be computed in a continuous space and not on a graph. Variants of the shortest path problem are subject to constraints that typically involves an upper limit on a function of the arcs. For instance, the goal can be to minimize the distance traveled by a vehicle with an upper bound on the travel time. This type of problem is usually expressed for one vehicle, for one path. It is an NP-hard problem for which some efficient methods have been developed [20–23].

Some other problems compute several shortest paths, *i.e.*, several vehicles are considered via a global criterion to be minimized. This is the case for the *traffic assignment problem* (TAP) which aims at reaching an equilibrium for the vehicles or for the whole system. The type of chosen equilibrium determines the objective function to be minimized. An example is the *Wardrop user and system equilibrium* [24, 25]. The problem is subject to flow conservation constraints and capacity constraints on arcs and, in the case of *system equilibrium*, it minimizes the average journey cost. This problem considers cooperation and possibly a decentralized management of the traffic. On the other hand, the *user equilibrium* is reached when no vehicle can lower its transportation cost through unilateral action. It is for instance used in road network applications, where roads do not have infinite capacity but the number of vehicles that can use each road in a given amount of time is limited.

The previously mentioned problems take into account the arcs of the graph to define routes, and possibly capacities on these routes. However, there are also other types of problems in which the grain is coarser: nodes are defined by geographical sectors, and a total capacity on each of these sectors is imposed.

This special case therefore involves capacities on the *vertices* of the graph. This type of problem appears in air transportation: it is then named *air traffic flow management problem* (ATFMP), originally defined in [26]. The objective function of the ATFMP is the total cost of aircraft delays, but variants can be derived by considering other objective functions. For example, the total cost of trajectories in terms of flight time, distance flown or CO_2 emitted can be taken into account. Optimization models addressing this problem typically involves the following decisions to be made for each flight:

- which sector to fly and when (which may induce speed modulations)?
- when to take off (by imposing delays with respect to the scheduled departure time)?

Various capacity upper bounds are imposed on:

- the total number of aircraft in each sector at any given time (sector capacity constraints),
- the total number of aircraft in each airport at any given time (airport capacity constraints).

This is a large-grained problem, but it is also necessary to take into account the more precise spatial scale of the arcs to know where to fly through a sector at a given time. This is done in the variant of the ATFM problem that involves rerouting (ATFMRP), and is solved in [27] (in its deterministic version). This problem is very detailed since it completely defines the trajectory followed, and it decides the speed of each aircraft on flown arcs. It takes into account a cost on each arc. This cost can be the flight time, the distance flown, or estimated CO_2 emissions on this arc, for each aircraft.

This paper proposes a method to organize traffic, while ensuring safety by satisfying capacity constraints on sectors. Moreover, it is also able to take into account contrails for minimizing the traffic environmental impact.

3 Mathematical optimization model

This section presents the mathematical optimization models for the subgraph--capacity multiple shortest path problem: Subsection 3.1 presents the static case while Subsection 3.2 addresses the time-dependent case.

3.1 Static case

This subsection focuses on the subgraph-capacity multiple shortest path problem in the static case (input data do not evolve with time).

The classical shortest-path problem on a graph involves only one vehicle and is modelled as follows. Let $G = (V, A)$ be a directed weighted graph, where V is the set of vertices, A is the set of arcs, and $w : A \to \mathbb{R}$ is the weight function. Let $s \in V$ be the start of the path, and $e \in V$ be the end of the path. Considering that the decision-variable vector X has a component $x_{u,v}$ for each arc $(u, v) \in A$, where $x_{u,v}$ indicates whether the arc (u, v) is part of the solution path or not, the optimization formulation of the shortest-path problem is:

$$\min_{X} \quad \sum_{(u,v)\in A} w_{u,v} x_{u,v} \tag{1a}$$

$$\text{s.t.} \quad \sum_{(u,v)\in A} x_{u,v} - \sum_{(v,u)\in A} x_{v,u} = 0,\ u \in V \backslash \{s, e\} \tag{1b}$$

$$\sum_{(s,v)\in A} x_{s,v} - \sum_{(v,s)\in A} x_{v,s} = 1 \tag{1c}$$

$$\sum_{(e,v)\in A} x_{e,v} - \sum_{(v,e)\in A} x_{v,e} = -1 \tag{1d}$$

$$X \in \{0,1\}^{|A|}, \tag{1e}$$

where (1b), (1c) and (1c) are the classical network flow conservation constraints (see for instance [15]) that ensure that the solution is a path from s to e (the incoming flow is equal to the outcoming flow on each vertex other than the source and the end).

This model can be adapted in the case of several vehicles with vehicle-specific weight functions, whose (u,v)-components are noted $w_{u,v,i}$, $(u,v) \in A$, $i = 1,2,\ldots,M$, where M is the number of vehicles, and for each vehicle $i = 1,2,\ldots,M$, a start vertex $s_i \in V$, and an end vertex $e_i \in V$ are given. Defining a decision-variable vector X_i for each vehicle, $i = 1,2,\ldots,M$, the optimization model for the multiple shortest-path problem then reads:

$$\min_{X} \quad \sum_{i=1}^{M} \sum_{(u,v)\in A} w_{u,v,i} x_{u,v,i} \tag{2a}$$

$$\text{s.t.} \quad \sum_{(u,v)\in A} x_{u,v,i} - \sum_{(v,u)\in A} x_{v,u,i} = 0, \quad u \in V \backslash \{s_i, e_i\}, \quad i = 1,2,\ldots,M \tag{2b}$$

$$\sum_{(s_i,v)\in A} x_{s_i,v,i} - \sum_{(v,s_i)\in A} x_{v,s_i,i} = 1, \quad i = 1,2,\ldots,M \tag{2c}$$

$$\sum_{(e_i,v)\in A} x_{e_i,v,i} - \sum_{(v,e_i)\in A} x_{v,e_i,i} = -1, \quad i = 1,2,\ldots,M \tag{2d}$$

$$X_i \in \{0,1\}^{|A|}, \quad i = 1,2,\ldots,M. \tag{2e}$$

Finally, a subgraph-capacity extension can be defined, provided a partition $\bigcup_{k=0}^{N} A_k = A$ of the set of arcs is given with corresponding capacities C_k, $k = 1,2,\ldots,N$, where N is the number of arc subsets considered. In the sequel, we shall call *sector* each of these arc subsets. We define an auxiliary decision-variable vector, Y_i, for each vehicle i, $i = 1,2,\ldots,M$, whose k^{th} component, $y_{k,i}$, indicates whether vehicle i uses arcs of sector A_k, $k = 1,2,\ldots,N$. The optimization model for the subgraph-capacity multiple shortest-path problem is then:

$$\min_{X,Y} \quad \sum_{i=1}^{M} \sum_{(u,v)\in A} w_{u,v,i} x_{u,v,i} \tag{3a}$$

$$\text{s.t.} \quad \sum_{(u,v)\in A} x_{u,v,i} - \sum_{(v,u)\in A} x_{v,u,i} = 0, \quad u \in V \backslash \{s_i, e_i\}, \quad i = 1,2,\ldots,M \tag{3b}$$

$$\sum_{(s_i,v)\in A} x_{s_i,v,i} - \sum_{(v,s_i)\in A} x_{v,s_i,i} = 1, \;\; i = 1,2,\dots,M \tag{3c}$$

$$\sum_{(e_i,v)\in A} x_{e_i,v,i} - \sum_{(v,e_i)\in A} x_{v,e_i,i} = -1, \;\; i = 1,2,\dots,M \tag{3d}$$

$$\sum_{i=1}^{M} y_{k,i} \leq C_k, \;\; k = 1,2,\dots,N \tag{3e}$$

$$y_{k,i} = 1 \text{ if and only if } \sum_{(u,v)\in A_k} x_{u,v,i} \geq 1, \;\; i = 1,2,\dots,M, \; k = 1,2,\dots,N \tag{3f}$$

$$X_i \in \{0,1\}^{|A|}, \;\; i = 1,2,\dots,M \tag{3g}$$

$$Y_i \in \{0,1\}^{N}, \;\; i = 1,2,\dots,M. \tag{3h}$$

Constraints (3b), (3c) and (3d) are the usual path flow conservation constraints for each vehicle, and constraints (3e) and (3f) are the new subgraph capacity constraints. Constraints (3f) enforce the definition of the auxiliary binary variables $Y_1, Y_2, \dots, Y_N$. These constraints can be linearized and the y_i's can be relaxed into continuous variables following the proposition 1:

Proposition 1. *Each of the constraints (3f), $i = 1,2,\dots,M$, can be replaced and linearized by:*

$$y_{k,i} \geq x_{u,v,i}, \;\; (u,v) \in A_k, \tag{4}$$

$$y_{k,i} \leq \sum_{(u,v)\in A_k} x_{u,v,i}, \tag{5}$$

$$y_{k,i} \in [0,1]. \tag{6}$$

Proof. Consider a sector k and a vehicle i. One can easily show that $1 - y_{k,i} = \prod_{(u,v)\in A_k} (1 - x_{u,v,i})$. Applying then the classical Fortet linearization extended to a product of several binary variables (see [28, Subsection 3.3.2]), one obtains:

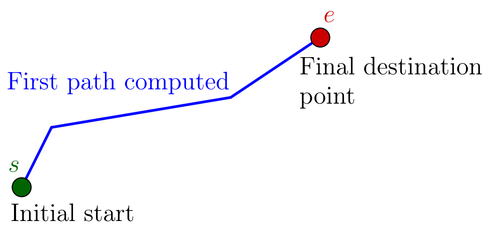
$$1 - y_{k,i} \leq 1 - x_{u,v,i}, \;\; (u,v) \in A_k,$$
$$1 - y_{k,i} \geq 1 - \sum_{(u,v)\in A_k} x_{u,v,i},$$
$$1 - y_{k,i} \in [0,1].$$

This yields straightforwardly the desired result. □

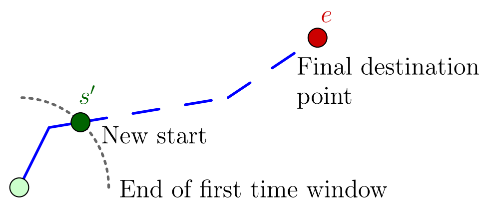
3.2 Time-dependent case

This subsection defines the mathematical formulation of the problem in the case where input data evolve with time. In the general case, we are considering time-dependent costs. Moreover, constraints (3e) and (3f) can also be time-dependent in case where a sector is only occupied by the vehicle during a certain amount of time and it is not occupied when the vehicle is not in the sector yet/anymore. The associated time-discretized model is detailed in Appendix A.

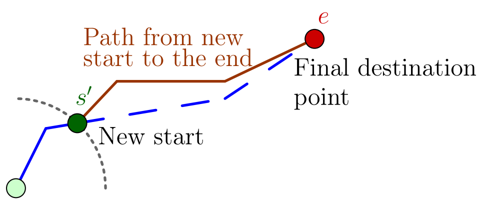
To take into account the release of the capacity of sectors by vehicles as they move, the paths are optimized for a succession of *(sliding) time windows*. More precisely, the paths on the graph are computed based on sector occupancy during the time interval under consideration. Then, the time is incremented by the sliding-window length Δt, and the start vertex s_i of each vehicle i is updated: it is replaced by the vertex reached by vehicle i in the previous sliding-window optimization. This process is illustrated by Figure 1. When, at the end of a time window, a vehicle is on an arc but not at a node, an artificial node is created. The process is stopped when each vehicle i has reached its final destination vertex e_i, $i = 1, 2, \ldots, M$. The entry time in the simulation of each vehicle is not artificially changed to coincide with the beginning of a time window. This time-window approach may lead to suboptimal solutions. It is used as a resolution heuristic for the following reasons. First, the time-dependent shortest path for a single vehicle is already a difficult problem [29], not to mention the case involving several vehicles plus capacity constraints. Moreover, in the context of our air transport application, the time-window methodology is tailored to operational concerns. Indeed, all relevant information is not always known for the subsequent time slots, and this time-sequential approach reduces drastically the uncertainties at each time window, by taking into account updated information.



(a) Compute the path from the starting point to the final destination point.



(b) Define the new starting point.



(c) Compute the path from the new starting point to the final destination point.

Fig. 1: Sliding window computation of a shortest path. The final path is computed sequentially, in pieces, for each of the time windows.

The paths on the graph are computed taking into account the capacity constraints only for a duration corresponding to K times the length of the sliding window, where K is a user-defined input parameter. This is illustrated by Figure 2. More precisely, constraints (3f) are modified as follows:

$$y_{k,i} = 1 \text{ if and only if } \sum_{(u,v)\in A_{k,K\,\Delta t}} x_{u,v,i} \geq 1, \ \begin{array}{l} i = 1, 2, \ldots, M, \\ k = 1, 2, \ldots, N, \end{array} \tag{7}$$

where $A_{k,K\,\Delta t}$ is the subset of arcs from A_k that have at least one end reachable by vehicle i within a time less than $K\,\Delta t$ from s_i.

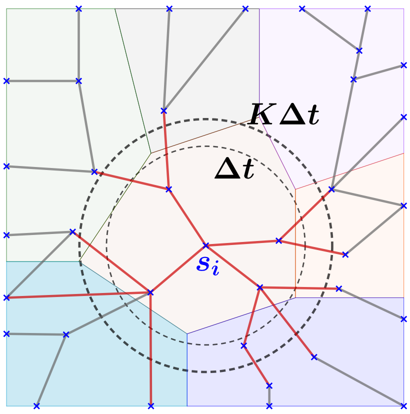


Fig. 2: Capacity computation along time windows. We consider that a vehicle i consumes capacity of the sectors that are reachable from the start point s_i within a time less than $K\,\Delta t$ (here: capacity is consumed only on red arcs).

4 Application to contrail avoidance and CO_2 minimization

Subsection 4.1 shows that contrail mitigation for several aircraft can be seen as an instance of the time-dependent subgraph-capacity multiple shortest path problem modeled in Section 3. Then, the cost function computation is detailed in Subsection 4.2, before focusing on the fairness issue in Subsection 4.3. Finally, Subsection 4.4 gives more explanations about the required input data.

4.1 General description of the application

In the sequel, the French upper airspace will be considered. Aircraft fly above France following a sequence of 3D or 2D points linked by straight-line segment routes. These points are named *waypoints*, and all the waypoints defined above France are the vertices of the graph. The segment routes defined between the waypoints define the set of arcs of the graph. The graph is thereby a directed graph, since aircraft routes are typically constrained to one direction. The problem of finding a trajectory for a flight therefore boils down to finding a path in a graph. The aim here is to minimize the global impact of all flights on the environment. More precisely, the goal is to minimize the total environmental cost while ensuring that the airspace capacities are not exceeded. The controller point of view is taken into account, and fairness among aircraft and airlines must be kept in mind. For this reason, and in order to avoid suboptimal solutions, a sequential (one aircraft at a time), greedy-like, computation of trajectories cannot be used, even though there are very efficient algorithms to solve single-vehicle shortest-path problems.

The problem is designed to compute only the *cruise* part of the trajectories. Indeed, departing and arriving an airport is subject to numerous extra operational constraints that leave almost no degree of freedom. Moreover, the altitudes concerned by the contrails are cruising altitudes which are quite high. It is therefore generally not relevant to address low-altitude parts of the trajectories. Then, the application we consider in this study is solved only in the 2D plane, *i.e.*, the altitude is not to be decided by the optimizer. The altitude is generally little modified during the cruise phase for reasons of passenger comfort. It is moreover already optimized by airlines to minimize fuel consumption and reduce engine worn out. Finally, the point of view chosen in this study is that of air traffic control. However, the weight of the aircraft being part of commercial data, it is not known by air traffic control. Without this data, it is not possible to know the ability of the aircraft to climb and to quantify precisely the environmental impact of a climb or a descent.

In the case of ground transportation, applications involving for instance cars on a road network, one can set a limited capacity per arc. In our air transportation application, the capacity limit is on areas (subsets of arcs), called *sectors*. These sectors represent subdivisions of the upper airspace, and more precisely a partition of the set of arcs. If the trajectories were to be computed sequentially, then some vehicles, the first ones, would be favored over the others. However, in the case of air transport, from the point of view of air traffic control, no airline should be favored over another, so such a sequential approach is not satisfactory. Above all, on the simple point of view of optimization, the greedy-like sequential approach is likely to lead to undesirable, suboptimal solutions.

Remark that satisfying capacity constraints allows one to ensure safety on the strategic time scale (several hours before the flights) that is considered here. It guarantees that the controllers will be able to manage the number of aircraft in their sector, while avoiding possible losses of separation in the shorter term.

Airspace capacities evolve with time and their occupancy evolve as aircraft enter or leave airspaces. Moreover, weather data, and also wind and contrail area data, evolve with time. The sliding-window approach is particularly adapted in this case for several reasons. Airspace capacities are defined on given time slots (in number of aircraft per time slot), and can be adapted from a time slot to another. It is also adapted for contrail area consideration since weather prediction are given for a period of time and are refined as time runs.

It should be noted that sectors can, in some cases, be *reconfigured* (sector grouping or splitting) to accommodate demand. In this study, the areas are considered static (the airspace configuration does not evolve with time).

4.2 Cost computation

As explained before, the goal here is to take into account CO_2 and non-CO_2 effects, and our application focuses on contrails. Bi-objective optimization is therefore a natural point of view to balance the two criteria that may be contradictory (burn more fuel - CO_2 - to avoid contrails - non-CO_2). However, the aim is to minimize the global environmental impact, and for this we use a metric that is common in the climate-change literature [14, 30, 31] to balance the two effects.

The cost, $w_{u,v,i}$ of an arc (u, v) for aircraft i is therefore the result of the (weighted) sum of the CO_2 and non-CO_2 costs: $w_{u,v,i} = w_{u,v,i}^{\mathrm{CO_2}} + w_{u,v,i}^{\text{non-CO}_2}$.

As mentioned before, the only non-CO_2 impact considered in this study is the contrails phenomenon: $w_{u,v,i}^{\text{non-CO}_2} = w_{u,v,i}^{\text{contrails}}$. Since the goal is to minimize the overall impact, it is necessary to quantify the relative impact of contrails versus that of CO_2. For this, a metric known as *Global Warming Potential* (GWP) is used. This metric relates the impact of most greenhouse gases to the impact of CO_2, under the form of a multiplicative factor, considering that the impact of CO_2 corresponds to GWP $= 1$. This metric depends on a time horizon over which the impact is computed. More details about GWP can be found in [32]. Table 1 gives different values of GWP for contrails according to the different considered time horizons, H. In the sequel, the *contrail-induced cirrus* (CIC) GWP will be noted g_H.

Table 1: *Global warming potential* for contrail for various time horizons, H.

	$H = 20$ years	$H = 100$ Years	$H = 500$ years
$\mathrm{GWP}_{\mathrm{contrail}}(H)$	0.74	0.21	0.064
$\mathrm{GWP}_{\mathrm{CIC}}(H)$	2.2	0.63	0.19

Depending on the time horizon, H, considered, the contrails have a more or less weight. Notably, they have less impact compared to CO_2 in the long term. In the following, only cirrus clouds induced by contrails will be taken into account, since they are the most impacting effect on the climate. However, other contrails can also easily be considered by our model, by a simple change in the cost function. Then, the cost for an arc (u, v) flown by aircraft i is: $w_{u,v,i} = (1 + \lambda_{u,v}\, g_H)\, w_{u,v,i}^{\mathrm{CO_2}}$, where $\lambda_{u,v} \in [0, 1]$ is the proportion of the arc (u, v) that lies is in a persistent contrail area.

Concerning the cost of CO_2, it is the total quantity of CO_2 emitted by the aircraft as it flies over the arc. The amount of CO_2 emitted per liter of standard jet fuel is constant. This cost is then directly proportional to the fuel consumption. Then, $w_{u,v,i}^{\mathrm{CO_2}} = C_{\mathrm{CO_2}}\, f_{u,v,i}\, t_{u,v,i}$, where $C_{\mathrm{CO_2}}$ is the constant quantity of CO_2 emitted by 1 kg of standard jet fuel, $f_{u,v,i}$ is the fuel flow of aircraft i above the arc (u, v), and $t_{u,v,i}$ its flight time. Then, the cost function reads:

$$w_{u,v,i} = (1 + \lambda_{u,v}\, g_H)\, C_{\mathrm{CO_2}}\, f_{u,v,i}\, t_{u,v,i}. \tag{8}$$

In the case studied, the aircraft evolve in the 2D plane and are assumed to not change speed. The fuel flow is then more or less constant and can be approximated by a representative fuel flow (given altitude and speed, representative mass of the cruise phase), f_i^r. Then, the cost function is rather wirtten as:

$$w_{u,v,i} = (1 + \lambda_{u,v}\, g_H)\, f_i^r\, t_{u,v,i}, \tag{9}$$

where multiplicative constants have been removed.

As mentioned earlier a sliding-window approach is adapted to the case of such weather-dependent cost functions, since there are uncertainties on weather forecast. In particular, contrails are difficult to predict, and their impact is even more difficult to predict [33]. Considering short-term path computation mitigates the uncertainties.

Details about how weather data are processed for the cost-function computation can be found in Appendix B.

4.3 Fairness consideration

In our application context, because of the air traffic control point of view, ensuring a certain level of fairness between aircraft and therefore between airlines is an issue that may emerge. The lack of fairness can occur on two points. First, satisfying airspace capacity constraints may result in detours for some aircraft and direct routes for others, and therefore some airlines are likely to be disadvantaged. Secondly, the avoidance of contrails has an economic cost for the airlines, and it is necessary to verify that an airline is not systematically penalized.

To address this issue, a penalty for flight time deviations from a reference flight time, noted t_i^{ref} for aircraft i, can be added to the objective function. Thus, the objective function becomes:

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{(u,v)\in A} w_{u,v,i}x_{u,v,i} + \mu \max_{i=1,\dots,N} \sum_{(u,v)\in A} \left| t_{u,v,i}x_{u,v,i} - t_i^{\mathrm{ref}} \right|, \tag{10}$$

where μ is a positive user-defined weighting parameter. The objective function then becomes non-linear, but can be linearized with classical reformulation techniques [28].

In this new objective function, the average cost is minimized as well as the penalty for the largest detour. The constant μ is used to give the desired relative importance to the worst-case penalty cost. The parameter t_i^{ref} is a reference time of flight for flight i. Its value can be, for example, the flight time when no capacity constraint is imputed and no contrail is taken into account if the user wants to consider the two criteria causing loss of fairness. On the one hand, if the user wants to avoid that the capacity constraints lead to loss of fairness, then t_i^{ref} can be set to the flight time without capacity constraint but with contrails taken into account. On the other hand, if the user wants to limit the impact of contrails on the loss of fairness, this reference time can be set to the flight time without consideration of the contrails but with consideration of the capacity constraints.

In this paper, it is chosen not to present results with this new objective function for two reasons. First, fairness is mainly taken into account by not computing the trajectories sequentially, in a greedy-like manner, but "all at once" to avoid systematically favoring some flights. Second, the goal is to show the possibility of reducing the impact of contrails at the network level, and to quantify these impacts. The results could therefore be distorted by taking fairness into account. In addition, other techniques can be used to ensure fairness rather than minimizing the impact of the largest detour (see for example [34]).

4.4 Data

This subsection details the input data of the problem. It explains in particular how the graph is built, and how the sectors are defined. The wind encountered and the areas favorable to contrails are also known data but the process to obtain the related information is detailed in Appendix B.

The present study follows the new principle of *Free Route Airspace* (FRA) which is applied nowadays to the European upper airspace. The aim of FRA is to remove the previously-established principle of air routes, and to replace it by navigation points, called waypoints, through which aircraft pass freely. A flight plan is therefore a simple sequence of waypoints through which the aircraft flies. This new paradigm allows one to consider an increased number of possible direct routes. As a consequence, the distance flown and thereby the CO_2 emissions, can be decreased.

Rules are still established to fly from a point to another even if there number aims to be decreased. Here, these rules are approximated by the rules established to build the graph $G = (V, A)$ required for the optimization model. The vertex set, V, is the set of considered waypoints. The arcs (set A) connect two waypoints when their inter distance is less than some user-defined threshold distance, $\overline{D}$. One could alternatively consider the complete graph, but this would not be coherent with operational practice, not to mention the increase of complexity in regards with the preliminary nature of the present study.

We construct instances based on the French waypoints that are located in the western and southern parts for now, see Figure 3 (the arcs are not displayed as they depend upon the maximum threshold distance, $\overline{D}$, chosen).

Our set of arcs is therefore:

$$A = \{(u, v) \mid u \in V, v \in V, d_{u,v} \leq \overline{D}\}, \tag{11}$$

where V is the set of waypoints, and $d_{u,v}$ is the distance between waypoints u and v.

For the subgraph capacities, the partition of the set A of arcs into N subsets, called sectors, $\{A_k\}_{k=1,2,...,N}$, is initially defined with respect to the waypoint set, as shown in different colors in Figure 4 for our instances. Then, all arcs with one of its ends in one such waypoint set, noted V_k for same $k = 1, 2, \dots, N$, is considered to be in the (arc-set) sector: $A_k = \{(u, v), u \in V_k \text{ or } v \in V_k\}$.

The next section reports computational results.

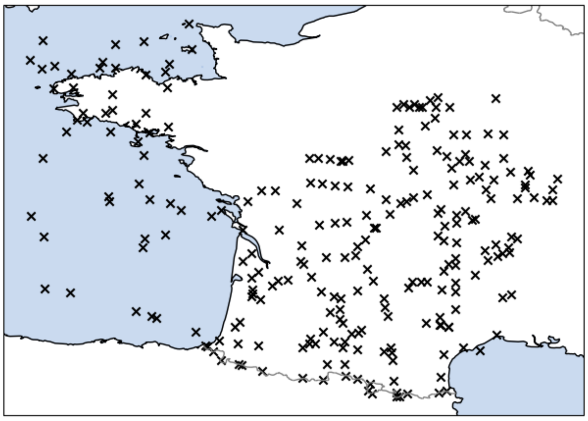


Fig. 3: The FRA waypoints above France constituting our instance set.

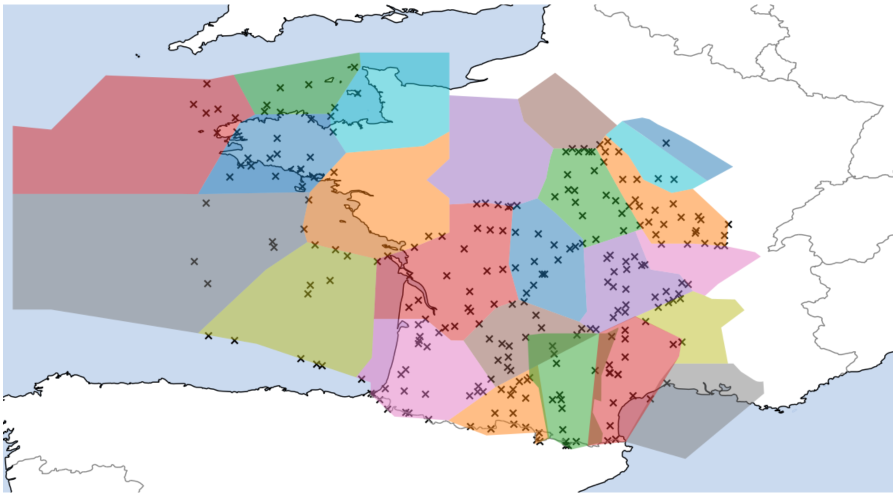


Fig. 4: The vertex subsets V_k's considered above France for building the arc set sectors A_k's for our instances.

5 Results and sensitivity analysis

This section presents an illustrative instance of the subgraph-capacity multiple shortest path problem together with various results obtained from numerical experiments resulting from a sensitivity analysis of the different parameters involved. An example of results is given in Subsection 5.1. Then, Subsection 5.2 focuses on the impact of wind on the results, while Subsection 5.3 addresses the impact of the time horizon chosen for the GWP computation on the results. Finally, the impact of the imposed airspace capacity, C_k, for sector A_k, $k = 1, 2, \ldots, N$, is discussed in Subsection 5.4.

5.1 Description of the illustrative instance

In the sequel, one instance of the problem is addressed but with various cost functions and various levels (right-hand side) for the capacity constraints. This subsection details the definition of this instance.

The graph is computed thanks to rules detailed in Section 4 (see Figures 3 and 4). Twenty aircraft are entering (taking off or entering the French upper airspace) per 30 minutes simultaneously, for three hours. The source-end vertex pair (s_i, e_i) of each aircraft i, $i = 1, 2, \dots, M$, are chosen randomly in the vertex set so that the minimum distance (as the crow flies) is 200 Nautical Miles and the resulting instance is named FRA-200. A Nautical Mile (NM) is the distance unit used in aeronautics, and corresponds to 1.852 km. In order to define the arc set, A, we set the maximum distance between two linked points at $\overline{D} = 75$ NM. The airspeed of all aircraft is set to 400 knots (a knot (kt) is the speed unit used in aeronautics and corresponds to 1 NM/h or 1.852 km/h). This speed is chosen in adequacy with the typical airspeed of a standard commercial aircraft, namely the Airbus A320 [35]. Then, to simplify the presentation, the fuel flow is considered the same for all the flights (identical aircraft types, engines and weights). Figure 5 displays the wind encountered, and Figure 6 shows the persistent contrails areas.

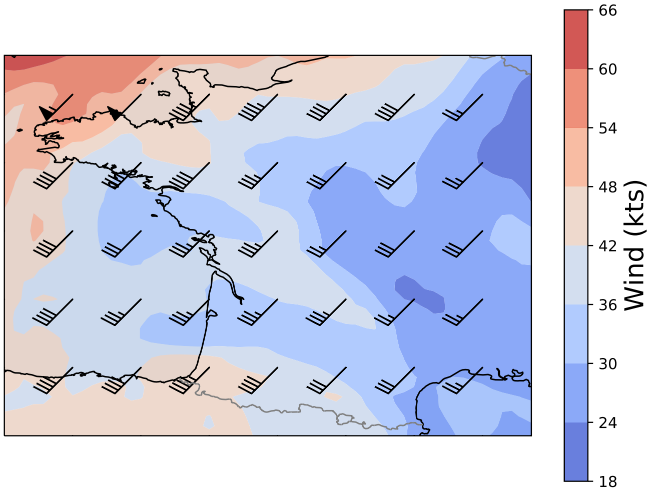


Fig. 5: Wind encountered in instance FRA-200.

The length of the sliding window for time-dependence consideration is set to $\Delta t =$ 15 minutes. The parameter K for capacity consumption (as defined in Equation (7)) is set to $K = 1.25$. If nothing else is explicitly mentioned, it is considered that:

- the capacity of each sector k is set to $C_k = 20,\ k = 1, 2, \dots, N$;
- the time horizon chosen for the GWP computation is set to $H = 100$ years.

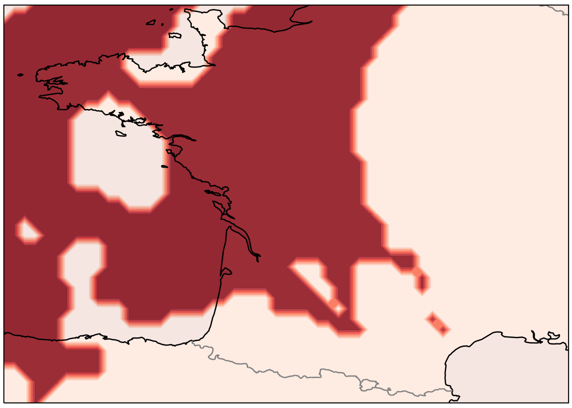


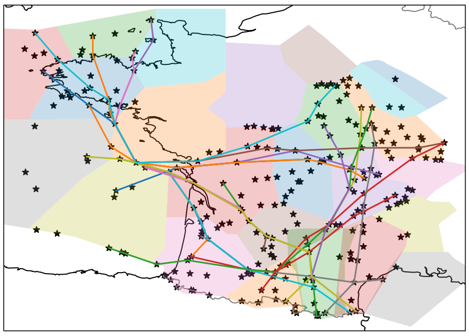
Fig. 6: Persistent-contrail areas used for instance FRA-200.

Table 2 summarizes the features of instance FRA-200. The details about the instance can be found in [36].

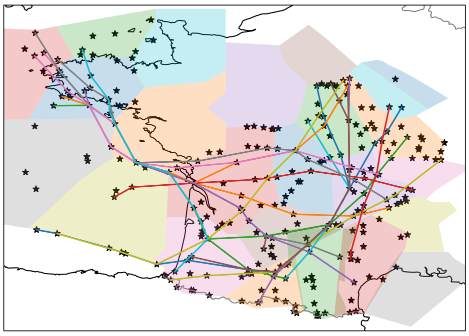
Table 2: Features of the illustrative instance, FRA-200.

Feature	**Notation**	**FRA-200**
Aircraft and trajectory features		
Number of aircraft	M	20 per 30 minutes for 3 hours
Airspeed of aircraft		400 kts
Minimum distance between s_i and e_i		200 NM
Airspace features		
Number of sectors	N	23
Maximum distance for arc definition	$\overline{D}$	75 NM
Capacity of each sector	C_k	20
Optimization model features		
Time-window length	Δt	15 minutes
Time horizon for the GWP computation	H	100 years

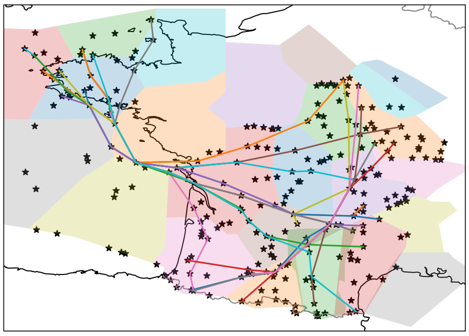
Instance FRA-200 is solved using the sliding window process explained in Subsection 3.2. The following results are obtained with the Java API of CPLEX on a computer with an Intel Core i5-10210U, 1.60 Hz, with 8 Go RAM and a Debian Linux OS. The code is available at [37]. Figure 7 shows the solution obtained on instance FRA-200 on maps. The trajectories are represented by groups of 20 aircraft on different maps for reading purposes. The computation time is 137.2 seconds.



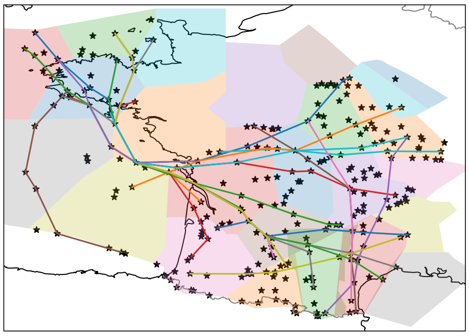
(a) Flights 0-19



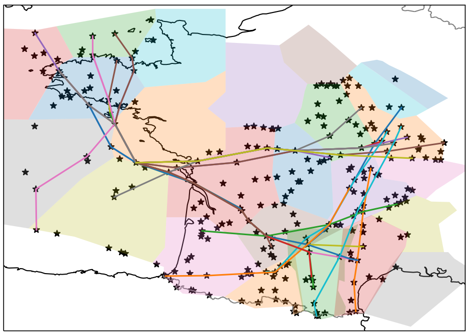
(b) Flights 20-39



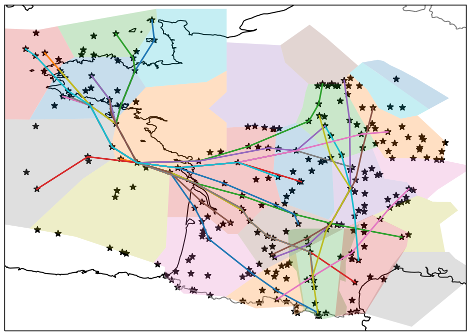
(c) Flights 40-59



(d) Flights 60-79



(e) Flights 80-99



(f) Flights 100-119

Fig. 7: Results obtained on instance FRA-200, grouped by 20 aircraft, sorted by increasing entry time.

5.2 Impact of wind

The wind has a certain impact on the results. To quantify this impact, two variants of instance FRA-200 have been solved:

1. without wind and without contrails: $w_{u,v,i} = d_{u,v},\ (u,v) \in A,\ i = 1, 2, \ldots, M$;
2. with wind and without contrails: $w_{u,v,i} = t_{u,v},\ (u,v) \in A,\ i = 1, 2, \ldots, M$.

Figure 8 displays the additional flight distance and flight time in the second case (with wind) in comparison to the first case (without wind).

The results show that in general, the flight distance is increased to the benefit of a reduced flight time, which was expected.

5.3 Impact of the time horizon used for GWP computation

As explained in Subsection 4.2, the cost factor associated with contrails depends on the chosen time horizon, H. Intuitively, the shorter the time horizon, the more impact the contrails have, and therefore the more beneficial it is to lengthen the trajectories to avoid contrails. To confirm this thought, three variants of instance FRA-200 are solved:

1. no contrail consideration: $g_H = 0$;
2. contrails consideration with $H = 100$ years: $g_H = g_{100} = 0.63$;
3. contrails consideration with $H = 20$ years: $g_H = g_{20} = 2.2$.

Figures 9 and 10 display comparative results for variants 1 and 2, and for variants 1 and 3, respectively.

The flight time is higher because of the avoidance of contrail area. Figures 9 and 10 show that the dependence on the time horizon is important since in the case of a short horizon, the optimal trajectories are much longer than in the case of a longer time horizon. If the contrails impact is considered high, the flight time has a lower

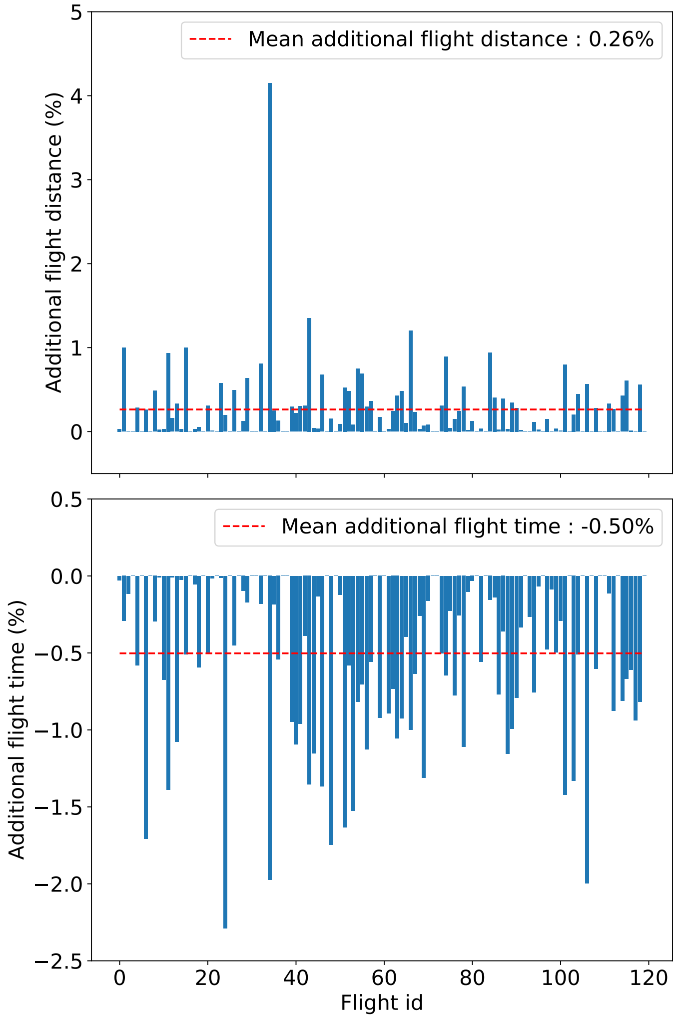


Fig. 8: Additional flight time and flight distance when wind is considered (flight time minimization) in comparison with results obtained when wind is not taken into account (only distance is minimized: $w_{u,v,i} = d_{u,v}$).

impact on the objective function, and then is highly increased to avoid as much as possible the areas favorable to contrails. Indeed, Figure 10 shows optimal trajectories with an increase of more than 30% in flight time. Fuel consumption therefore explodes. From an environmental point of view, it is not necessarily relevant to choose such a trajectory. It is also economically costly (increase in fuel and extra cost associated to flight delays). Finally, such an extra consumption has also an impact on safety since a significant part of the extra fuel that every aircraft must carry in order to anticipate a possible diversion to another airport will be used for avoiding contrails.

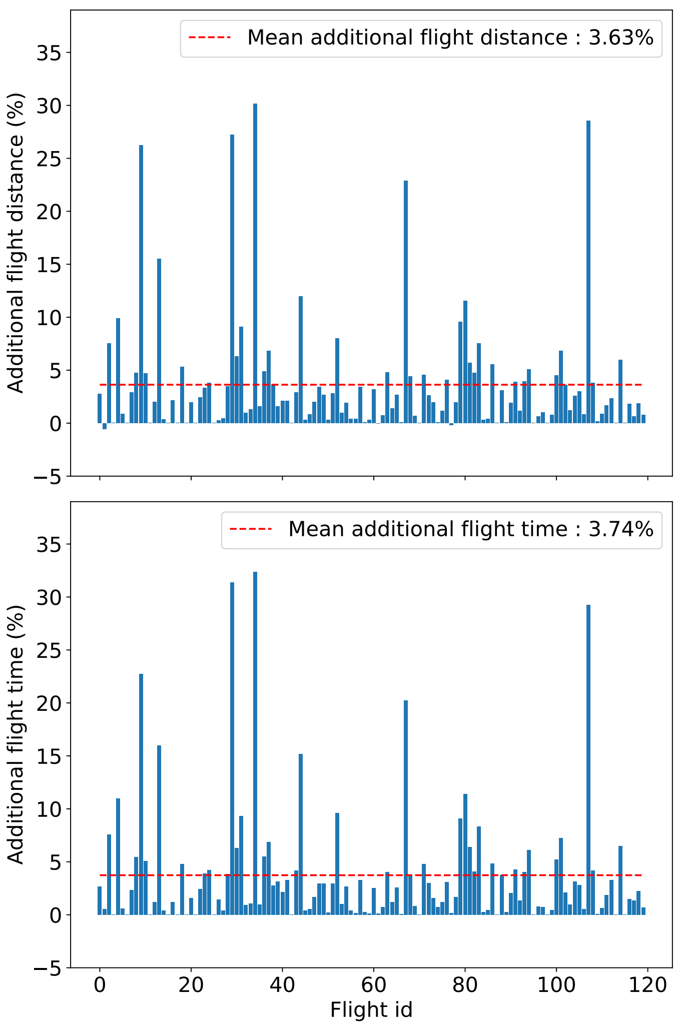


Fig. 9: Additional flight time and flight distance when $H = 100$ years ($g_H = g_{100} = 0.63$) when compared with the case without contrails ($g_H = 0$).

An extra constraint should therefore be added to our optimization model to avoid such undesirable solutions. It is interesting to consider adding other extra constraint so as to satisfy other operational needs. For example, if an airline is planning connections between its flights, it is desirable that the flights involved are not delayed too much so that the connection is guaranteed.

Let $t_{0,i}$ be the flight time without any contrail consideration. It can be computed thanks to the optimization process with $g_H = 0$. We propose to add to the model

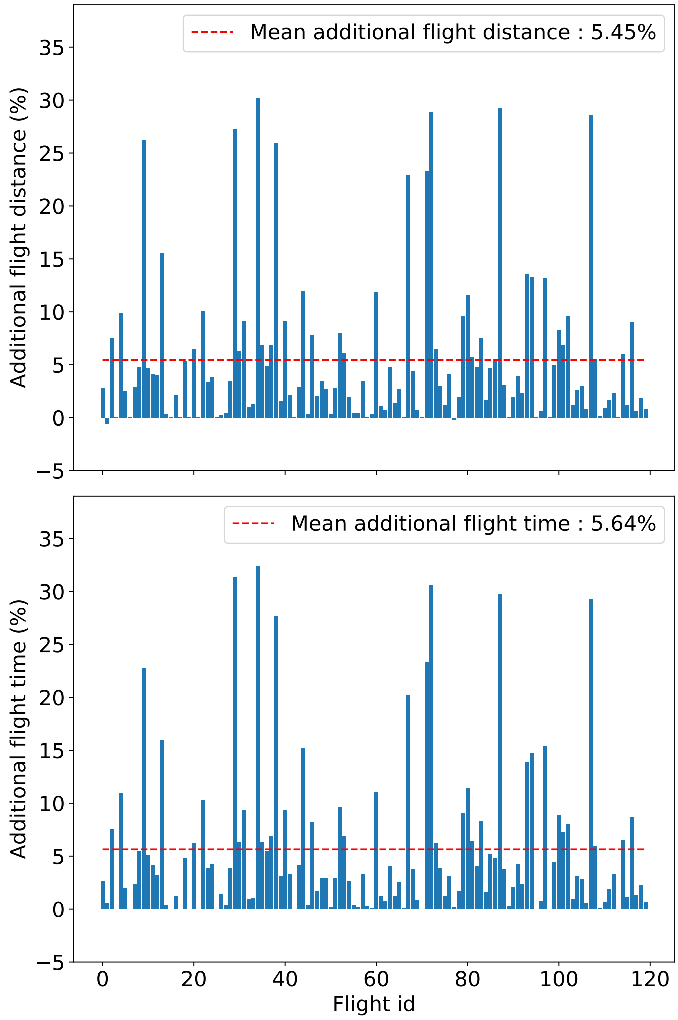


Fig. 10: Additional flight time and flight distance when $H = 20$ years ($g_H = g_{20} = 2.2$) when compared with the case without contrails ($g_H = 0$).

constraints

$$\sum_{(u,v)\in A} t_{u,v,i}\ x_{u,v,i} \leq C\ t_{0,i},\ \ i = 1, 2, \ldots, M, \tag{12}$$

where $t_{u,v,i}$ is the flight time over the arc (u, v) for the flight i, and where C is a user-defined constant to control the additional flight time allowed.

Figure 11 shows an example of results obtained when such constraints are added with $C = 1.1$, (10% of additional flight time is allowed).

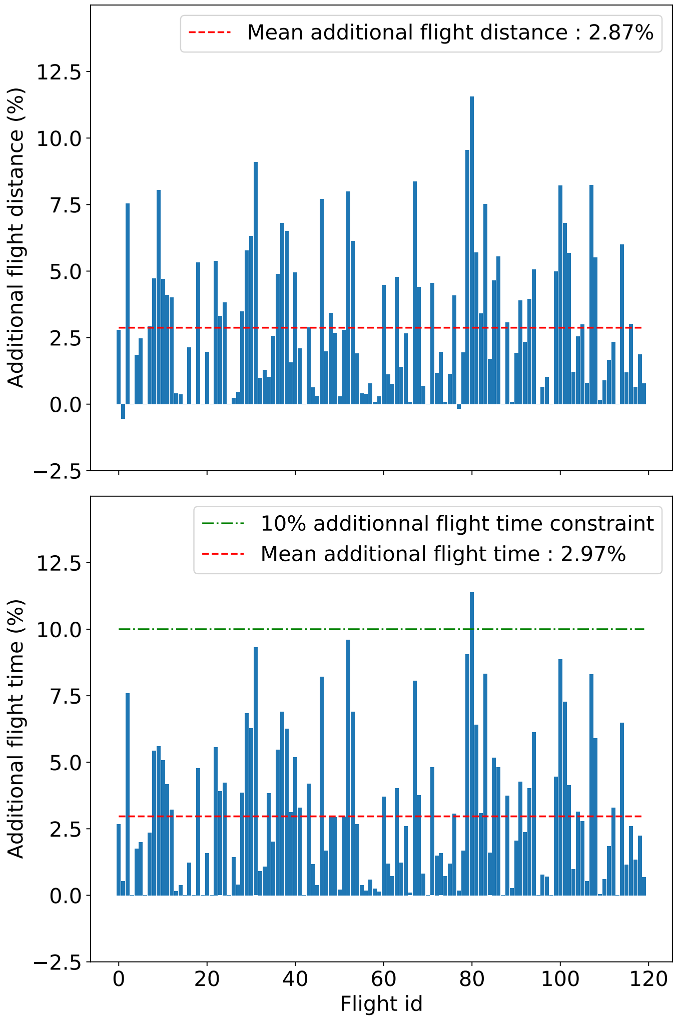


Fig. 11: Additional flight time and flight distance with extra flight time constraints added when $H = 20$ years, in comparison with the case without contrails.

Because of the sliding-window process, the extra constraints are not globally satisfied, but they are satisfied locally at each step, and are nearly satisfied at the global scale, as can be observed in Figure 11.

5.4 Impact of airspace capacity

A last parameter to be studied is the airspace capacity values C_k's. The same setup concerning aircraft is taken with a GWP computed with, this time, $H = 100$ years. Several results are impacted by changing the airspace capacity C_k, $k = 1, 2, \ldots, N$, especially the obtained optimal objective-function value, and the computation time.

Indeed, in order to satisfy capacity constraints, some aircraft may be forced to fly longer or through contrail zones. The problem can be harder to solve if the capacity constraints are restrictive, and so the computation time increases. Figure 12 shows the evolution of the optimal objective-function value and the computation time obtained for various capacity levels, C_k, assumed constant for every sector k, $k = 1, 2, \ldots, N$, and at each time window.

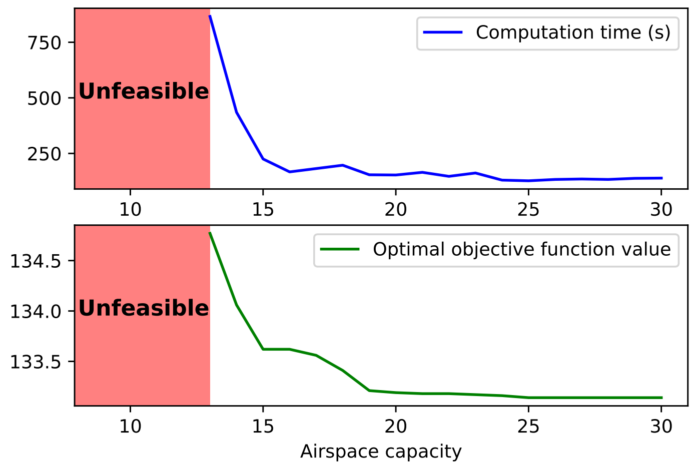


Fig. 12: Result comparison in terms of objective-function value and computation time when the airspace capacity, C_k, changes.

The above results show how the right-hand side of the airspace capacity constraints affect the feasibility of the problem and the environmental impact. The higher the capacity of the airspace is, the better the results are. However, beyond a certain value of C_k, no change is observed in computation time, nor in objective-function value since the capacity constraints are not saturated.

6 Conclusion

Motivated by the issue of contrail-avoidance at the network scale from the air traffic control point of view, we introduced a new model for the multiple shortest-path problem that takes into account capacity constraints on subgraphs. In the static case, the number of vehicles going through each considered subgraph is counted and bounded. Dynamic aspects have also been taken into account by establishing an adapted model with time windows over each of which the static model is solved. Taking this time-dependent aspect into account is essential because as vehicles move, they free up capacity, and costs can also change over time. The model used does not provide the same quality of solution as if the static model had been discretized in time, but it fits better the operational reality as illustrated by the application addressed in our study. Moreover, the proposal methodology allows one to use a standard solver to solve realistic air traffic instances.

The application addressed in this paper motivated the introduction of a new model for the contrail-avoidance problem when considered at the network scale and from the air traffic control point of view. The time-window strategy is particularly adapted to model time-dependence since it corresponds to operational realities (time between contrail forecasts and uncertainty on their prediction, airspace capacities defined per time period, etc.). The trajectories are computed for all aircraft and not for each individual aircraft to avoid favoring any particular flight. Work has also been done for computing cost of arcs, so that they can be adapted to the Air Traffic Management point of view. This allows one to consider and to weight CO_2 and contrails impact using Global Warming Potential with different time horizons, giving more or less importance to contrail avoidance. Moreover, it allows one to take into account other non-CO_2 effects due to air transport, or to use more elaborated metrics to determine the impact of contrails.

Future tracks of research should focus on the numerous sources of uncertainty to be taken into account when addressing the contrail-avoidance problem. There are several uncertainties to consider when studying contrail-favorable (or persistent-contrail-favorable) areas, as shown by Gierens *et al.* [33]. Other source of uncertainty are the wind estimation and the presence of traffic.

Moreover, when considering the air traffic control point of view, one faces another crucial criterion: fairness for which ideas have been evoked in this paper. Future work should adapt the proposed model so as to quantify and enforce fairness between airlines.

Finally, other decision variables could be considered. First, in this study, the airspace configuration is considered static, the sectors are not modified. However, reconfiguration may also be involved in decisions to accommodate the location of contrail areas. Also, the cruise altitude should be envisaged as decision variables since it is an efficient mean of mitigating contrail impact, as shown by Fichter *et al.* [38].

This leads to several issues including the knowledge of some critical aircraft parameters (such as its weight at any moment) which are not always known from the air traffic control point of view. Moreover, fuel consumption depends on the altitude as well as on the type of aircraft. Models to estimate fuel flows from an air traffic control perspective should be considered. For that, the OpenAP [39] database could be used, or other models based on machine learning as it has been done in [40] for the approach and landing phases.

Supplementary information. Not applicable.

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Ethics approval/declarations. Not applicable.

Consent to participate. Not applicable.

Consent for publication. Not applicable.

Availability of data and materials. Sample data are available at: https://cloud.recherche.enac.fr/index.php/s/i6jxDFM8GnSAgyF.

Code availability. Code is available in the following git repository: http://celine.demouge@git.recherche.enac.fr/time_dependent_contrails_avoidance_at_network_scale.git.

Authors' contributions. CD: Methodology, Software, Validation, Formal analysis, Writing. MM: Methodology, Validation, Formal analysis, Writing. NC: Methodology, Validation, Formal analysis, Writing. DD: Methodology, Validation, Formal analysis, Writing. All authors reviewed the manuscript.

Appendix A Time-discretized optimization model

The mathematical optimization model obtained in the static case can be discretized to obtain the time-dependent optimization model. This transformation is based on the classical *time-dependent shortest path* problem [41].

Some notations should be defined first:

- $T_i = \{t_{0,i}, \ldots, t_{f,i}\}$: the set of time slots for vehicle i;
- $T = \bigcup_{i=1}^{M} T_i$;
- $C_{k,t}$: the capacity of sector k, $k = 1, 2, \ldots, N$, for time slot t, $t \in T$;
- $w^t_{u,v,i}$: the cost for vehicle i, $i = 1, 2, \ldots, M$, to go through arc $(u, v) \in A$ at time slot t, $t \in T_i$;
- $\Delta^t_{u,v,i}$: the time necessary for vehicle i, $i = 1, 2, \ldots, M$, to go through arc $(u, v) \in A$ at time slot t, $t \in T_i$.

The decision variables are for each vehicle i, $i = 1, 2, \ldots, M$:

- $x_{u,v,i} \in \{0, 1\}$ is equal to 1 if vehicle i goes through arc (u, v);
- $z^t_{u,v,i} \in \{0, 1\}$ is equal to 1 if vehicle i enters arc (u, v) at time slot t;
- $y^t_{k,i} \in \{0, 1\}$ is equal to 1 if vehicle i flies through sector k at time slot t, $t \in T$.

The model is then:

$$\min_{X,Y,Z} \quad \sum_{i=1}^{M} \sum_{t \in T_i} \sum_{(u,v) \in A} w^t_{u,v,i} z^t_{u,v,i} \tag{A1a}$$

$$\text{s.t.} \quad \sum_{(u,v) \in A} x_{u,v,i} - \sum_{(v,u) \in A} x_{v,u,i} = 0, \ u \in V \backslash \{s_i, e_i\}, \ i = 1, 2, \ldots, M \tag{A1b}$$

$$\sum_{(s_i,v)\in A} x_{s_i,v,i} - \sum_{(v,s_i)\in A} x_{v,s_i,i} = 1,\ i = 1,2,\dots,M \tag{A1c}$$

$$\sum_{(e_i,v)\in A} x_{e_i,v,i} - \sum_{(v,e_i)\in A} x_{v,e_i,i} = -1,\ i = 1,2,\dots,M \tag{A1d}$$

$$\sum_{(u,v)\in A} z^t_{u,v,i} - \sum_{(v,u)\in A} z^{t+\Delta^t_{v,u,i}}_{v,u,i} = 0,\ u \in V\backslash\{s_i,e_i\},\ i = 1,2,\dots,M,\ t \in T_i \tag{A1e}$$

$$\sum_{(s_i,v)\in A} z^{t_{0,i}}_{s_i,v,i} = 1,\ i = 1,2,\dots,M \tag{A1f}$$

$$\sum_{t\in T_i} z^t_{u,v,i} = x_{u,v,i},\ (u,v) \in A,\ k = 1,\dots,N,\ i = 1,\dots,M \tag{A1g}$$

$$\sum_{i=1}^{M} y^t_{k,i} \leq C_{k,t},\ k = 1,2,\dots,N,\ t \in T \tag{A1h}$$

$$y^t_{k,i} = 1 \text{ if and only if } \sum_{(u,v)\in A_k} z^t_{u,v,i} \geq 1,\ i = 1,2,\dots,M,\ k = 1,2,\dots,N,\ t \in T_i \tag{A1i}$$

$$y^t_{k,i} = 0,\ i = 1,2,\dots,M,\ k = 1,2,\dots,N,\ t \in T\backslash T_i \tag{A1j}$$

$$X_i \in \{0,1\}^{|A|},\ i = 1,2,\dots,M \tag{A1k}$$

$$Z_{i,t} \in \{0,1\}^{|A|},\ i = 1,2,\dots,M,\ t \in T_i \tag{A1l}$$

$$Y_{i,t} \in \{0,1\}^{N},\ i = 1,2,\dots,M,\ t \in T_i. \tag{A1m}$$

Constraints (A1b),(A1c) and (A1d) are the flow conservation constraints. Constraints (A1e) and (A1f) enforce consistency of space-time flow conservation. Constraints (A1g) make the link between the time-dependent and the static decision variables. Constraints (A1h) are the time-discretized capacity constraints. Constraints (A1i) and (A1j) define the auxiliary variables $y^t_{k,i}$. The former can easily be linearized, as for the static model using Proposition 1.

Appendix B Data processing for numerical experiments

This section shows how data for computational experiments have been extracted and computed. Subsection B.1 deals with wind data, while Subsection B.2 focuses on contrail data.

B.1 Wind data

The costs defined by (9) involves the computation of the flight time over each arc $(u, v) \in A$. To compute these costs, the wind on the arcs, and the distance between u and v, the two ends of the arcs, have to be known.

Let λ_u and λ_v be the latitude of vertices u and v respectively, and let ϕ_u and ϕ_v be their longitude. The distance between u and v is given by:

$$\begin{aligned} d_{u,v} &= R\ c_{rad} \ \text{ in km}, && \text{(B2)} \\ &= 60\ c_{degrees} \ \text{ in NM}, && \text{(B3)} \end{aligned}$$

where $R = 6,371$ km is the Earth radius, c_{rad} and $c_{degrees}$ represent the following c values, expressed in radians and in degrees respectively:

$$c = \arccos\Big(\sin(\lambda_u)\ \sin(\lambda_v) + \cos(\lambda_u)\ \cos(\lambda_v)\ \cos(\phi_v - \phi_u)\Big). \tag{B4}$$

Then, the flight time for aircraft i, noted $t_{u,v,i}$, between points u and v is given by:

$$t_{u,v,i} = \frac{d_{u,v}}{GS_{u,v,i}}, \tag{B5}$$

where $GS_{u,v,i}$ is the ground speed of aircraft i on arc (u, v). It can be computed via:

$$GS_{u,v,i} = V_{a_i} + W_{u,v}, \tag{B6}$$

where V_{a_i} is the airspeed of aircraft i (considered constant), and $W_{u,v}$ is the wind encountered on the arc (u, v).

Wind data have been extracted from the website Windy [42] on a square grid of size 0.2 degree above France, as shown on Figure B1.

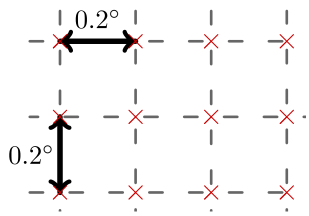

Fig. B1: 2D grid used for wind data extraction from Windy [42].

To compute the wind on each node of the graph, a so-called Shepard interpolation [43] was used. More precisely, for each node P located in a 2D-square $P_1P_2P_3P_4$ of the data grid (see Figure B2), the wind $W(P)$ at P is calculated from the wind at P_k, $k = 1, 2, 3, 4$, using the distance from P to each of these points (noted respectively d_1, d_2, d_3 and d_4) as follows:

$$W(P) = \frac{\sum_{i=1}^{4} W(P_i) * d_i^{-p}}{\sum_{i=1}^{4} d_i^{-p}}, \tag{B7}$$

where $p > 1$ is a user-defined parameter (set to $p = 2$ in this study).

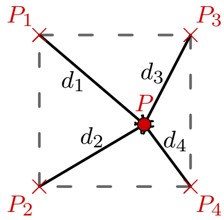

Fig. B2: Notations for estimating the wind at P via Shepard interpolation.

Finally, the wind along an arc (u, v) is simply defined as the average of that at u and at v:

$$W_{(u,v)} = \frac{W(u) + W(v)}{2}. \tag{B8}$$

Figure B3 shows data used for the examples presented in the result section (Section 5).

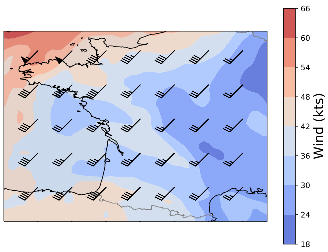

Fig. B3: Wind encountered in the example of the result.

B.2 Contrail data

Contrails are formed in cold and humid areas. They persist and induce cirrus if the air is supersaturated in ice. The computation of persistent contrail areas is performed in two phases:

1. Areas favorable to contrail formation
2. Areas in which contrails will persist (ice supersaturated areas).

In [31], contrails areas are computed thanks to the *Schmidt-Appleman criterion*. This criterion gives a minimum threshold, r_{min}, of relative humidity of the air in liquid water, noted RH_w, above which contrails are formed: contrails are assumed to form when $RH_w \geq r_{min}$, where

$$r_{min} = \frac{G(T - T_c) + e_{sat}^{liq}(T_c)}{e_{sat}^{liq}(T)}, \tag{B9}$$

$e_{sat}^{liq}(T)$ is the saturation vapor pressure over water, and T_c is the estimated threshold temperature (in Celsius degrees) for contrail formation at liquid saturation. The later is computed via:

$$T_c = -46.46 + 9.43 \log(G - 0.053) + 0.72 \log^2(G - 0.053), \tag{B10}$$

where $G = \frac{EI_{H_2O} C_p P}{\epsilon Q(1-\eta)}$, $EI_{H_2O} = 1.25$ is the water vapor emission index, $C_p = 1004\ J.kg^{-1}.K^{-1}$ is the heat capacity of the air, P is the ambient pressure (in Pascals), $\epsilon = 0.6222$ is the ratio of the molecular masses of water and dry air, $Q = 43 * 10^6 J.kg^{-1}$ is the specific heat of combustion, and $\eta = 0.3$ is the average propulsion efficiency of a commercial aircraft.

In [31], the ice super saturated areas are determined thanks to the following criterion: $RH_i > 1$, where the relative humidity over the ice, noted RH_i is computed as follows:

$$RH_i = RH_w * \frac{6.0612 * \exp(\frac{18.102*T}{249.52+T})}{6.1162 * \exp(\frac{22.577*T}{273.78+T})}, \tag{B11}$$

and where T is the ambient temperature in Celsius degrees.

The relative humidity and temperature are also computed from data extracted from Windy [42] on a 2D-grid, and interpolated via quadratic interpolation. Figure B4 shows the data used for the examples presented in the result section (Section 5), where red areas are persistent-contrail-favorable areas, to be avoided.

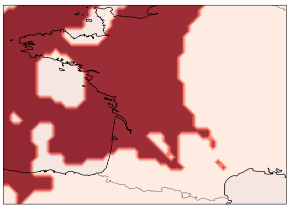

Fig. B4: Persistent-contrail areas in western France used for our instances.

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